

Time-varying preferences for storable goods*

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Abstract

We investigate a storable good market where a firm faces consumers with time-varying preferences that differ between the purchase and consumption stage. Consumers hold (possibly biased) beliefs at the purchase stage. The quantity purchased for storage — i.e., ex ante storage — stimulates current sales, whereas the quantity actually stored — i.e., ex post storage — depresses future sales. A trade-off emerges such that the firm may prefer price cuts to promote ex ante storage with increasing demand and price rises to dampen ex post storage with decreasing demand. We provide novel insights into the puzzling phenomenon of countercyclical pricing. Empirically, we test these predictions in the vitamin market and other product categories, finding robust evidence of countercyclical pricing. The observed patterns can be attributed to consumer naivete and uncertainty in demand, consistent with the model's key mechanisms.

KEYWORDS: countercyclical pricing, ex ante storage, ex post storage, naïve consumers, rational consumers, time-varying preferences, sophisticated consumers.

JEL CLASSIFICATION: D21, D42, L12.

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1 Introduction

A large number of products traded in upstream or downstream markets are perishable in consumption but can be stored for future consumption. The presence of consumer storage has been empirically documented in the economic literature, especially in anticipation of higher future prices (e.g., Hendel and Nevo 2004, 2006a, 2006b; Osborne 2018; Perrone 2017; Pendorfer 2002; Pires 2016; Wang 2015). This suggests that firms selling storable goods must devote significant attention to future changes in market conditions. Natural examples studied in the literature are intertemporal fluctuations in market demand (Dudine et al. 2006) or in production costs (Antoniou and Fiocco 2023).

In this paper, we investigate a storable good market where a firm faces consumers whose preferences vary over time. Departing from the extant literature, we consider time-varying preferences that differ between the purchase stage and the consumption stage. For a wide range of goods, consumers can only form beliefs about their tastes at the time of purchase and learn their actual valuation at the time of consumption. This is typically the case of experience goods, which can often be stored for future consumption, such as wine, medications, health care, and beauty products. Furthermore, as emphasized in the literature on consumer naïveté, some consumers hold biased beliefs about their preferences and revise their decisions over time, typically because of present-focused preferences, preference reversals, and time inconsistency (e.g., Ericson and Laibson 2019; Laibson 1997; O'Donoghue and Rabin 1999; Read and van Leeuwen 1998; Strack and Taubinsky 2024). These patterns are common in markets for hedonic goods, where consumers may underestimate or overestimate their future consumption levels. Consumers' decisions can be also affected by exogenous shocks, such as changes in weather conditions. This occurs with various storable goods, including vitamins, canned soup, and soft drinks.

Our work aims to contribute to the economic literature in different aspects. We design an analytically tractable framework for storable goods where consumer preferences vary between the purchase stage and the consumption stage. This allows us to fully characterize a firm's dynamic pricing policy and consumers' storage incentives. Our methodologically challenging approach provides novel insights into the effects of storability on price dynamics in a setting that accounts for consumers' (possibly biased) beliefs about their preferences. As it will be clear later, the predictions of our model enrich those arising from a classical storable good

setting à la Dudine et al. (2006), where demand evolves over time but consumer preferences are invariant between the purchase stage and the consumption stage and thus buyers do not revise their decisions after purchasing the good. Allowing for storability, our model also identifies a richer dynamic context for the study of consumers' biases. On a more applied standpoint, as discussed below, our results square with well-documented different pricing patterns and shed new light on the puzzling phenomenon of countercyclical pricing.

We consider a general framework where consumers hold beliefs about their valuation for a storable good at the purchase stage and learn their actual valuation at the consumption stage. Time-varying preferences for storable goods imply that the quantity purchased for storage, referred to as *ex ante storage*, may depart from the quantity actually stored, referred to as *ex post storage*. The firm benefits from *ex ante storage*, which stimulates current sales. Conversely, *ex post storage* is detrimental to the firm because it depresses future sales. The identification of *ex ante* and *ex post storage* crucially relies upon a condition for intertemporal marginal utilities from consumption for each buyer. At any positive level of *ex post storage* this condition becomes binding and dictates the equalization of marginal utilities over time. Remarkably, the condition for intertemporal marginal utilities is directly satisfied for all consumers (as it corresponds to the storability constraint) in a classical model à la Dudine et al. (2006) where consumer preferences are invariant between the purchase and the consumption stage, which implies that *ex ante storage* coincides with *ex post storage*. In our framework, this is no longer the case and thus time-varying preferences introduce novel effects into the firm's dynamic problem. When promoting *ex ante storage*, the firm benefits from further consumption by buyers that discover high valuation for the good. However, buyers with low valuation accumulate in the form of *ex post storage* not only the entire amount of *ex ante storage* but also the difference between their expected and actual demand. As a result, the firm faces a trade-off when promoting *ex ante storage*.

To suitably identify the different forces behind this trade-off, we first examine a setting characterized by consumer naïveté. A fraction of the consumer population consists of 'sophisticated' consumers that hold unbiased beliefs and perfectly know their preferences at the purchase stage. The remaining fraction is formed by 'naïve' consumers that hold biased beliefs at the purchase stage and learn their actual valuation at the consumption stage. Naïfs may either underestimate or overestimate their preferences when purchasing the good.¹ With

¹Our results qualitatively carry over to a setting where underestimating consumers coexist with overestimating

naïve underestimating consumers, the firm gains from promoting ex ante storage because it anticipates that naïfs shall consume a larger quantity after learning their high valuation for the good. Thus, ex ante storage translates into further consumption. The presence of consumers that become aware of their stronger preferences inflates future demand and leads to prices rising over time. However, the promotion of ex ante storage is costly for the firm because sophisticates accumulate this amount in the form of ex post storage, which is consumed without being bought at a higher price in the future. Ex ante storage arises for sufficiently small values of the storage cost. The firm reduces the price below the static monopoly level to encourage ex ante storage, especially as long as static monopoly prices are feasible. Conversely, when naïve consumers overestimate their preferences, ex ante storage is detrimental to the firm because it cannot induce further consumption. Demand falls in the future and prices decline over time. As consumers do not have any incentives to buy for storage purposes in anticipation of lower future prices, ex ante storage disappears. However, naïfs still prefer to store ex post by keeping some leftovers for future consumption after learning low valuation for the good, provided that the storage cost is sufficiently small. The condition for intertemporal marginal utilities is binding in equilibrium and thus naïfs' ex post storage is such that their marginal utilities are equalized over time. To dampen ex post storage that depresses future sales, the firm raises the price above the static monopoly level.

Equipped with the results under consumer naïveté, we turn to a setting where all consumers are rational and hold unbiased beliefs but they are uncertain about their preferences at the purchase stage. This allows us to endogenize the firm's choice between allowing ex ante storage (which leads to ex post storage as well) and only ex post storage. Under consumer naïveté, this choice is crucially driven by whether naïfs underestimate or overestimate their preferences. Furthermore, with naïve underestimating consumers, only sophisticates store ex post and their levels of ex ante and ex post storage coincide. With naïve overestimating consumers, ex ante storage vanishes. With rational consumers, however, the levels of ex ante and ex post storage are simultaneously determined in a non-trivial manner. Whereas consumers with high valuation for the good are induced to consume the entire quantity bought, the condition for intertemporal marginal utilities is binding in equilibrium for consumers with low valuation and thus the amounts of ex ante and ex post storage equalize their marginal utilities

consumers. In Section 4, we provide examples of markets where consumers may underestimate or overestimate their preferences.

over time. We show that the firm prefers to stimulate ex ante storage when future demand is sufficiently high. Otherwise, the firm opts for allowing only ex post storage. Interestingly, we find that allowing only ex post storage can be more profitable for the firm even when ex ante storage is feasible. As ex ante storage stimulates current sales and ex post storage depresses future sales, this result might appear *prima facie* surprising. To gain some insights, it is helpful to note that allowing only ex post storage reduces *ceteris paribus* the quantity bought and stored for future consumption. Furthermore, as prices tend to decrease over time, buyers are inclined to consume the good to a larger extent rather than to store it. The equilibrium price dynamics and the associated distortions from the static monopoly level resemble those derived under consumer naïveté. Although consumers are ex ante identical, the firm implements ex post discrimination by favoring consumption by buyers with high valuation for the good.

In a setting à la Dudine et al. (2006) with increasing demand over time, the firm raises the price above the static monopoly level to dampen consumer storage that harms the firm's profit because consumers use the quantity stored instead of buying the good at a higher price in the future. With time-varying preferences, our model predicts that the firm may prefer price cuts to promote storage with increasing demand and price rises to dampen storage with decreasing demand. Endogenizing dynamic pricing and consumer storage, we identify a novel and complementary explanation for the systematic empirical evidence of countercyclical pricing — namely, price decreases (increases) during high (low) demand season — for a large variety of storable goods, such as beer, cheese, crackers, tuna, and canned soup (e.g., Butters et al. 2025; Chevalier et al. 2003; Guler et al. 2014; Haviv 2022; Kwon et al. 2023; MacDonald 2000; Meza and Sudhir 2006; Nevo and Hatzitaskos 2006). Our model is rich enough to accommodate for procyclicality in prices as well, in line with some empirical observations (e.g., Butters et al. 2025; Gijsenberg 2017). In Section 7, we provide a stylized empirical application of our model for different storable product categories, which suggests some evidence in support of our theoretical findings. Differently from previous empirical studies, we aim to reconcile the demand-side and the supply-side perspectives and identify the role of the magnitude of demand.

We conduct our investigation in a fairly general and flexible framework that allows for extensions in different directions. In Section 6, we show that our main results carry over to an arbitrary (finite or infinite) number of periods. We also extend the model to nonlinear pricing and firm's full commitment to future prices. As argued in Section 8, our work delivers some

potentially significant empirical predictions and policy implications.

The rest of the paper is organized as follows. Section 2 describes the related literature. Section 3 characterizes the general framework. Section 4 considers the setting with consumer naïveté. Section 5 turns to the setting with rational consumers. Section 6 examines different extensions. Section 7 provides a stylized empirical application of our model. Section 8 concludes the analysis. The Appendix collects the main formal proofs. The Supplementary Appendix shows additional formal results and associated proofs.

2 Related literature

Our paper belongs to the extensive literature on storable goods. Early influential works about the effects of storability on dynamic pricing include Bénabou (1989) in an inflationary environment and Hong et al. (2002) with shoppers and captive consumers. Our study is closely related to the seminal paper of Dudine et al. (2006), which considers a storable good monopoly market where demand fluctuates deterministically over time and, differently from our framework, consumer preferences do not change between the purchase and consumption stage. Hendel and Nevo (2013) examine intertemporal price discrimination across consumers with heterogeneous storage abilities. In a model à la Dudine et al. (2006), Antoniou and Fiocco (2019) identify a firm's strategic incentives to hold inventories under limited commitment. With intertemporal cost variations, Antoniou and Fiocco (2023) analyze the effects of demand curvature and consumer storage cost on price dynamics. Hendel et al. (2014) develop a seminal model of nonlinear pricing of storable goods. Bhaskar and Roketskiy (2024) demonstrate that, differently from Hendel et al. (2014), nonlinear prices allow firms to screen different inventory levels under transitory monopoly power. The economic effects of storability have also been studied in different competitive environments. Anton and Das Varma (2005) consider quantity competition, whereas Guo and Villas-Boas (2007) turn to price competition. Nava and Schiraldi (2014) examine the sustainability of collusion. Mitraile and Thille (2009, 2014) explore speculators' competitive storage under various market structures where production is either monopolistic or imperfectly competitive.

Our work can also contribute to the fast-growing literature on consumer naïveté. Starting from the pioneering work of Strotz (1955) about individuals' time inconsistency, consumer naïveté has been studied from different angles. Laibson (1997) considers consumers with hy-

perbolic discount functions. O'Donoghue and Rabin (1999) characterize naïfs' procrastination of immediate-cost activities and preproperation of immediate-reward activities. Eliaz and Spiegler (2006) derive the menu of contracts that the principal offers to screen the agent's sophistication. Heidhues and Kőszegi (2009) investigate costly yet futile attempts at self-control and find that higher sophistication often decreases welfare. Christensen and Nafziger (2016) derive the equilibrium package sizes of a sinful product when consumers have self-control problems and face shopping costs. Heidhues and Kőszegi (2017) study naïveté-based price discrimination and find that the exploitation of consumers' mistakes often reduces welfare. Heidhues et al. (2021) examine the regulation of secondary features of complex products when consumers have limited attention. Heidhues and Strack (2021) identify present bias from the timing of choices in a setting where a partially naïve quasi-hyperbolic discounter repeatedly chooses whether to complete a task. Strack and Taubinsky (2024) establish the conditions for the identification of time inconsistency in empirical designs. Triviza (2024) derives the optimal pricing scheme when consumers are naïve about their addiction to a good. We refer to Ericson and Laibson (2019) for an excellent review of intertemporal choice, especially in the presence of present-focused preferences and preference reversals. Consumer naïveté has also been experimentally and empirically estimated. Read and van Leeuwen (1998) find that the participants in a field experiment exhibit present-focused preferences because they choose far more unhealthy snacks for immediate choice than for advance choice. More recently, the existence of present bias has been documented by Acland and Levy (2015) and Fedyk (2024). DellaVigna and Malmendier (2004) show that the profit maximizing contract design with partially naïve time inconsistent consumers matches the features of several industries.

Our paper is also related to the vast literature on durable goods. Storable goods differ from durable goods because they are perishable in consumption but can be stored for future consumption. The literature on durable goods has traditionally considered rational consumers. A seminal contribution is Board (2008) about the monopolist's profit maximizing strategy with incoming demand that evolves over time. Nocke and Peitz (2003) is the first contribution that introduces hyperbolic discounting in a durable good market. Departing from the previous strands of literature, we investigate the effects of storability on price dynamics when consumers hold (possibly biased) beliefs about their preferences that vary between the purchase and consumption stage, which provides novel insights into the puzzling phenomenon of countercyclical pricing.

3 General framework

Consumers. We consider a two-period market for a storable good characterized by a continuum of consumers normalized to unity. Consumers have either high or low valuation for the good. Preferences of consumer type $k = h, l$ are represented by a standard quasi-linear utility function $U_k(x_\tau, y_\tau) = u_k(x_\tau) + y_\tau$, where $u_k(x_\tau)$ is utility from the consumption level x_τ and y_τ is money in period $\tau \in \{1, 2\}$. At the first period purchase stage, consumers are uncertain about their preferences for the good. They assign a probability $\theta \in [0, 1]$ of having high utility $u_h(x_\tau)$ and a complementary probability $1 - \theta$ of having low utility $u_l(x_\tau)$ from the consumption level x_τ in period $\tau \in \{1, 2\}$. For each consumer type $k = h, l$, the continuously differentiable function $u_k(x_\tau)$ is increasing and concave in x_τ , i.e., $u'_k(x_\tau) > 0$ and $u''_k(x_\tau) < 0$, with the standard normalization $u_k(0) = 0$. Furthermore, it holds $u'_h(x_\tau) > u'_l(x_\tau)$ for any consumption level x_τ . At the first period consumption stage, consumers learn their actual valuation for the good. The static demand function $D_k(p)$ associated with utility $u_k(\cdot)$ is given by $D_k(p) \triangleq \arg \max_q \{u_k(q) - qp\}$, $k = h, l$, which is continuously differentiable and decreasing with the price p , i.e., $D'_k(p) < 0$. Furthermore, it holds $D_h(p) > D_l(p)$ for any price p (as $u'_h(\cdot) > u'_l(\cdot)$). Let $D(p) \triangleq \arg \max_q \{\theta u_h(q) + (1 - \theta) u_l(q) - qp\}$ be the static demand function stemming from the consumer expected utility maximization problem, which is continuously differentiable and decreasing with the price p , i.e., $D'(p) < 0$. As in the first period consumers are uncertain about their valuation for the good, the first period demand for consumption $D_1(p_1)$ at the purchase stage corresponds to the static demand function $D(p_1)$. In the second period, consumers know their valuation for the good and thus the second period demand for consumption $D_2(p_2)$ consists of high demand $D_h(p_2)$ for the share θ of consumers with high valuation and low demand $D_l(p_2)$ for the residual share $1 - \theta$ of consumers with low valuation. As shown in Section 5, we also allow for a general formulation of $D_2(\cdot)$, which may include exogenous shocks, and our results are unaffected if consumer preferences change across periods. Thus, $D_2(\cdot)$ may stem from the consumer expected utility maximization problem when consumers are uncertain about their preferences in the second period as well. The timing of the game is summarized at the end of the section. As discussed in the introduction, in Section 4 we examine a setting where a fraction of the consumer population consists of naïve consumers that exhibit biased beliefs about their preferences. In Section 5, we turn to a setting where all consumers are rational and hold unbiased beliefs.

Consumers can store the good in the first period for consumption in the second period at a unit cost $c \geq 0$. Without loss of insights, there is no discounting on the second period. We denote by *ex ante storage* the amount of the good that consumers buy for storage purposes at the first period purchase stage before learning their valuation for the good. In line with the main literature (e.g., Anton and Das Varma 2005; Dudine et al. 2006), for any given first period price p_1 and second period expected price p_2^e , the level of ex ante storage for each consumer (equal across all consumers) is given by

$$s(p_1, p_2^e) = \begin{cases} D_2(p_1 + c) & \text{if } p_1 + c < p_2^e \\ [0, D_2(p_1 + c)] & \text{if } p_1 + c = p_2^e \\ 0 & \text{if } p_1 + c > p_2^e. \end{cases} \quad (1)$$

As shown below, the last two cases are the only relevant outcomes in equilibrium.² Throughout the paper, we refer to $p_1 + c \geq p_2$ as the *storability constraint*.³

We denote by *ex post storage* the level of storage that consumers actually carry to the second period after learning their valuation for the good. For any given first period price p_1 and second period expected price p_2^e , the level of ex post storage $\bar{s}_k(p_1, p_2^e)$ per consumer of type $k = h, l$ crucially depends on intertemporal marginal utilities from consumption. Specifically, given the quantity $D_1(p_1) + s(p_1, p_2^e)$ purchased in the first period, the level of consumption per consumer of type $k = h, l$ satisfies the following *condition for intertemporal marginal utilities*

$$u'_k(D_1(p_1) + s(p_1, p_2^e) - \bar{s}_k(p_1, p_2^e)) + c \geq u'_k(D_k(p_2^e)). \quad (2)$$

The left-hand side of condition (2) captures the first period marginal utility from consumption, which is inflated by the storage cost c forgone for each unit consumed. The right-hand side of condition (2) represents the second period marginal utility from consumption, which corresponds to the second period expected price, i.e., $u'_k(D_k(p_2^e)) = p_2^e$. Intuitively, in equilibrium the marginal utility from consumption in the first period cannot be lower than the marginal utility from consumption in the second period. Otherwise, consumers would be eager to increase the amount of ex post storage up to the level such that no consumption takes place in the first period and thus the marginal utility from consumption becomes higher in the first

²In the proof of Lemma 1 (see the Appendix), we provide the derivation of ex ante storage $s(p_1, p_2^e)$ in (1).

³For expositional convenience, we sometimes write p_2 instead of p_2^e , because the expected price coincides with the equilibrium price in the second period.

period (as $u_k''(\cdot) < 0$). Notably, in a classical storable good setting à la Dudine et al. (2006) where demand changes over time but consumer preferences are invariant between the purchase and consumption stage, ex ante storage coincides with ex post storage and the condition for intertemporal marginal utilities (2) boils down to the storability constraint $p_1 + c \geq p_2$. As we show in the subsequent analysis, time-varying preferences introduce novel effects into the firm's dynamic problem.

When condition (2) holds for any $\bar{s}_k(p_1, p_2^e) \geq 0$, consumers of type $k = h, l$ prefer to consume the entire quantity bought in the first period and thus they do not store ex post, i.e., $\bar{s}_k(p_1, p_2^e) = 0$. Otherwise, condition (2) is binding in equilibrium and ex post storage equalizes consumers' marginal utilities over time. As the second period marginal utility is equal to the corresponding price, i.e., $u_k'(D_k(p_2^e)) = p_2^e$, the level of ex post storage $\bar{s}_k(p_1, p_2^e)$ per consumer of type $k = h, l$ satisfies the following *condition for the equalization of intertemporal marginal utilities*

$$\bar{s}_k(p_1, p_2^e) = D_1(p_1) + s(p_1, p_2^e) - D_k(p_2^e - c). \quad (3)$$

Consumers of type $k = h, l$ store the difference between the quantity purchased $D_1(p_1) + s(p_1, p_2^e)$ and the quantity consumed $D_k(p_2^e - c)$. As consumers may have incentives to revise their consumption and storage decisions after learning their preferences, ex post storage $\bar{s}_k(p_1, p_2^e)$ may differ from ex ante storage $s(p_1, p_2^e)$.

Firm. A monopolistic firm operates in the market. Let $\Pi \triangleq \Pi_1 + \Pi_2$ be the firm's aggregate profit, where the first period profit Π_1 and the second period profit Π_2 are respectively given by

$$\Pi_1 = p_1 [D_1(p_1) + S(p_1, p_2^e)] \quad \text{and} \quad \Pi_2 = p_2 [D_2(p_2) - \bar{S}(p_1, p_2^e)], \quad (4)$$

where $S(p_1, p_2^e)$ denotes the aggregate ex ante storage and $\bar{S}(p_1, p_2^e)$ represents the aggregate ex post storage. The first period demand faced by the firm consists of the demand for consumption $D_1(p_1)$ inflated by ex ante storage $S(p_1, p_2^e)$. In the second period, the firm faces a demand equal to the demand for consumption $D_2(p_2)$ reduced by ex post storage $\bar{S}(p_1, p_2^e)$ because consumers use this quantity to satisfy their consumption needs. Due to the lack of commitment to future prices, the firm maximizes its continuation profit in each period. Production costs are normalized to zero. The profit function Π_τ in period τ satisfies the standard

assumption of profit concavity, i.e., $\Pi''_{\tau}(p_{\tau}) < 0$, $\tau \in \{1, 2\}$, which ensures that the second-order conditions for profit maximization are fulfilled.

We can identify three pricing policies at the firm's disposal. The first pricing policy is such that the storability constraint is strictly satisfied, i.e., $p_1 + c > p_2$. This removes consumers' storage incentives and leads to the static monopoly prices, provided that they can be implemented, as described below. The second pricing policy makes the storability constraint binding, i.e., $p_1 + c = p_2$. Consumers are indifferent between storing the good in the first period and purchasing it in the second period. The third pricing policy, i.e., $p_1 + c < p_2$, induces consumers to buy in order to store the entire demand in the second period. Yet, this pricing policy is not feasible because the firm succumbs to the temptation to reduce the price in the second period, which enhances its profit through higher sales.⁴

Given the firm's aggregate profit $\Pi \triangleq \Pi_1 + \Pi_2$, where Π_1 and Π_2 are described in (4), the firm's dynamic profit maximization problem writes as

$$\max_{\{p_1, p_2\}} \Pi(p_1, p_2). \quad (5)$$

For any level of ex post storage $\bar{S}(p_1, p_2^e) \geq 0$ inherited from the first period, the firm's dynamic profit maximization problem in (5) is subject to the following *sequential optimality constraint*

$$p_2(\bar{S}(p_1, p_2^e)) \triangleq \arg \max_{\tilde{p}_2} [D_2(\tilde{p}_2) - \bar{S}(p_1, p_2^e)]. \quad (6)$$

If the storability constraint is binding, i.e., $p_1 + c = p_2$, the sequential optimality constraint (6) and the condition for intertemporal marginal utilities (2) jointly determine the amount of ex ante storage $S(p_1, p_2^e)$ and the amount of ex post storage $\bar{S}(p_1, p_2^e)$. If the storability constraint is not binding, i.e., $p_1 + c > p_2$, the sequential optimality constraint (6) identifies the second period price p_2 and the condition for intertemporal marginal utilities (2) dictates the level of ex post storage $\bar{S}(p_1, p_2^e)$, whereas ex ante storage vanishes, i.e., $S(p_1, p_2^e) = 0$.

Timing and equilibrium concept. The first period of the game exhibits the following three stages. First, the firm sets the price for the good. Second, consumers make their purchase decisions. Third, consumers learn their actual valuation for the good and consumption takes place. The quantity purchased but not consumed is stored for future consumption. In the second period, the game evolves as in the first period, with the only difference that all consumers

⁴We refer to Lemma 1 (in the Appendix) for technical details.

know their actual preferences. The solution concept is the subgame perfect Nash equilibrium.

4 Naïve consumers

We first examine a setting characterized by consumer naïveté. A fraction $\lambda \in (0,1)$ of the consumer population consists of ‘sophisticated’ consumers and the remaining fraction $1 - \lambda$ is formed of ‘naïve’ consumers. Consumers of type $i = s, n$ (where s stands for sophisticates and n for naïfs) may have either high or low valuation for the good. Sophisticated consumers hold unbiased beliefs about their preferences and perfectly know their valuation for the good at the purchase stage. Naïve consumers hold biased beliefs about their preferences. Specifically, at the first period purchase stage, naïfs can either underestimate or overestimate their preferences when purchasing the good. We identify two main classes of naïve consumers according to the direction of their belief biases.

Underestimating consumers: Naïfs underestimating their preferences exhibit high utility $u_h(x_\tau)$ but at the first period purchase stage believe that they have low utility $u_l(x_\tau)$ for any consumption level x_τ .

Overestimating consumers: Naïfs overestimating their preferences exhibit low utility $u_l(x_\tau)$ but at the first period purchase stage believe that they have high utility $u_h(x_\tau)$ for any consumption level x_τ .

Underestimating consumers anticipate lower utility than the actual one for any consumption level, whereas overestimating consumers expect higher utility than what eventually materializes. As stated in Section 3, it holds $u'_h(\cdot) > u'_l(\cdot)$. We also suppose that $|u''_h(\cdot)| \geq |u''_l(\cdot)|$.⁵ For the sake of convenience, we consider the situation where at the first period purchase stage naïfs believe that they share the same valuation for the good as sophisticates. Thus, consumers are ex ante identical but ex post heterogeneous.⁶ We disentangle our analysis according to naïfs’ belief biases.

⁵This ensures that the difference between the static demand functions $D_h(p)$ and $D_l(p)$ does not decrease with the price p . Such an assumption allows for a sharper characterization of our results and is imposed only for technical tractability.

⁶Our results qualitatively carry over to a more complicated setting where naïfs also differ ex ante from sophisticates. As it will be clear below, what matters for our analysis is that (at least) some consumers exhibit preferences that vary between the purchase stage and the consumption stage. In line with the literature on consumer naïveté, despite holding biased beliefs about their valuation for the good at the first period purchase stage, naïve consumers can correctly anticipate what happens in the second period. This is because naïfs are aware that a fraction of consumers will discover the actual valuation only at the consumption stage but believe that they belong to the fraction that already knows it at the purchase stage.

As mentioned in the introduction, consumers may misjudge their preferences for hedonic goods (e.g., Acland and Levy 2015; Fedyk 2024; Read and van Leeuwen 1998). Consumers underestimating their preferences exhibit a cognitive bias defined as ‘cold-to-hot empathy gap’. They are in a calm, non-aroused state and fail to anticipate their future consumption of some products, once a strong impulse or visceral state arises. Typical examples include snacks, beverages, alcohol, and tobacco. Conversely, consumers overestimating their preferences experience a cognitive bias defined as ‘hot-to-cold empathy gap’. Driven by momentary craving or discomfort, they exaggerate their future desire for some products, such as indulgent desserts, oversized portions, and comfort remedies.

4.1 Underestimating consumers

Naïve consumers underestimating their preferences have high utility $u_h(\cdot) \triangleq u_n(\cdot)$ but at the first period purchase stage believe that they share the same low utility $u_l(\cdot) \triangleq u_s(\cdot)$ as sophisticates. As all consumers are convinced of having utility $u_s(\cdot)$ at the first period purchase stage, the first period demand for consumption is given by $D_1(\cdot) \triangleq D_s(\cdot)$, where $D_1(\cdot)$ (described in Section 3) is evaluated at $\theta = 0$. Furthermore, consumers choose the same level of ex ante storage in (1), namely, the amount of the good bought for storage purposes and intended for future consumption before their actual preferences are realized. In the second period, all consumers are aware of their actual preferences and the demand for consumption becomes $D_2(p_2) \triangleq \lambda D_s(p_2) + (1 - \lambda) D_n(p_2)$. Ex post storage $\bar{S}(p_1, p_2^e)$ may differ from ex ante storage $S(p_1, p_2^e)$ because naïve consumers adjust their consumption and storage decisions after learning their actual preferences.

When naïve consumers underestimate their preferences, demand grows in the second period because naïfs learn their high valuation for the good. This implies that the first period static monopoly price $p_1^{sm} \triangleq \arg \max_{p_1} p_1 D_s(p_1)$ is lower than the second period static monopoly price $p_2^m \triangleq \arg \max_{p_2} p_2 D_2(p_2)$. The static solution is implementable under the possibility of storage as long as consumers do not have any (strict) incentives to store, i.e., $p_1^{sm} + c \geq p_2^m$. There exists a threshold $\tilde{c}^u \triangleq p_2^m - p_1^{sm} > 0$ for the storage cost c such that the static solution is implementable if and only if it satisfies the following *static feasibility constraint*

$$c \geq \tilde{c}^u. \tag{7}$$

The storage cost c must be sufficiently high to dissuade consumers from buying in the first period in order to store for consumption in the second period at the static monopoly prices. Otherwise, consumers are inclined to store and the static solution cannot be achieved. A larger fraction λ of sophisticates (with low valuation for the good) reduces the second period demand $D_2(\cdot)$ and leads to a lower second period static monopoly price p_2^m . Thus, a higher λ decreases the threshold \tilde{c}^u and relaxes the static feasibility constraint (7).

The firm's dynamic problem in (5) is affected by the possibility of storage at least when the static feasibility constraint (7) fails to hold. As a lower storage cost makes consumers more eager to store, we find that there exists a threshold $\bar{c}^u > 0$ for the storage cost c below which storage can emerge in equilibrium.⁷ The dynamic storage solution must satisfy the following *dynamic feasibility constraint*

$$c < \bar{c}^u. \tag{8}$$

As formalized in Lemma 1 (see the Appendix), in the dynamic storage solution, prices are linked through the binding storability constraint, i.e., $p_2 = p_1 + c$, which makes consumers indifferent about storage. Although consumers behave identically at the first period purchase stage, the firm can benefit from storage because it anticipates that naïve consumers shall discover high valuation for the good and prefer to consume in the first period more than what they initially planned. However, this comes at the firm's cost of forgoing c for each unit sold at p_1 and stored in the first period rather than at $p_2 = p_1 + c$ in the second period. The firm can adjust the first period price to manipulate storage through the sequential optimality constraint (6). Specifically, the firm can promote storage by cutting the first period price. A reduction in the first period price leads to a lower price in the second period as well (due to the binding storability constraint), which requires a higher level of storage to dampen the second period demand net of storage so that the sequential optimality constraint (6) is satisfied. Notably, the firm prefers to set prices such that sophisticates store some quantity for future consumption, whereas naïfs do not store ex post. Any price that induces naïfs to carry some quantity to the second period would be unduly low for the firm, which would lose c for each unit sold at p_1 rather than at $p_2 = p_1 + c$ in the second period. In other terms, ex post storage is $\bar{s}_h(p_1, p_2^e) \triangleq \bar{s}_n(p_1, p_2^e) = 0$ per naïve consumer and $\bar{s}_l(p_1, p_2^e) \triangleq \bar{s}_s(p_1, p_2^e) > 0$ per sophisti-

⁷We refer to Lemma 1 (in the Appendix) for technical details.

cated consumer. Thus, the condition for intertemporal marginal utilities (2) is (generally) slack in equilibrium for naïfs. However, this condition is binding in equilibrium for sophisticates and reflects the condition for the equalization of intertemporal marginal utilities (3). This coincides with the binding storability constraint because the levels of ex ante and ex post storage are the same for sophisticates.⁸

Given the static feasibility constraint (7) and the dynamic feasibility constraint (8), we find that for $\tilde{c}^u < c < \bar{c}^u$ the static solution and the dynamic storage solution are both implementable. As formalized in Lemma 2 (see the Appendix), this interval is non-empty if and only if the fraction λ of sophisticated consumers is below a certain threshold, i.e., $\lambda < \tilde{\lambda}$. The idea is that, when the number of sophisticates is small enough, storage becomes particularly profitable for the firm because a large portion of consumers is naïve and consumes in the first period the entire quantity bought. In Lemma 3 (see the Appendix), we characterize the solution that the firm implements in equilibrium. There exists a threshold $\hat{c}^u \in [\tilde{c}^u, \bar{c}^u]$ such that for $c < \hat{c}^u$ the dynamic storage solution arises in equilibrium, despite the feasibility of the static solution. Intuitively, the firm promotes storage as long as its cost is relatively low. Otherwise, the static solution emerges in equilibrium. For $\lambda > \tilde{\lambda}$, the static solution and the dynamic storage solution are mutually exclusive. The firm implements the dynamic storage solution whenever it is feasible, i.e., $c < \bar{c}^u$.

In light of these results, we can now characterize the equilibrium price dynamics with underestimating consumers. We focus on the most relevant case where the storage cost is sufficiently low that storage emerges in equilibrium. A full characterization is provided in the proof of Proposition 1.

Proposition 1 *In the first period, the equilibrium price in the presence of storage exhibits the following features: (i) for $c < \tilde{c}^u$, it is higher than the static monopoly price if $\lambda > \bar{\lambda}$, and (ii) for $c \geq \tilde{c}^u$, it is lower than the static monopoly price. In the second period, the equilibrium price in the presence of storage is lower than the static monopoly price.*

Proposition 1 shows that in the first period the equilibrium dynamic pricing crucially differs from the static monopoly level according to the magnitude of the storage cost and the

⁸As shown in Lemma 1 (see the Appendix), when the storage cost is high enough, i.e., $c \geq \bar{c}^u$, the dynamic storage solution cannot arise because the dynamic feasibility constraint (8) is violated and thus storage vanishes. This leads to the dynamic solution without storage, where prices are still linked through the binding storability constraint but the second period price is at the static monopoly level, as implied by the sequential optimality constraint (6).

number of sophisticates. When the storage cost is sufficiently low that the static feasibility constraint (7) is violated, i.e., $c < \tilde{c}^u$, the firm raises the first period price above the static monopoly level if the number of sophisticates is relatively large, i.e., $\lambda > \bar{\lambda}$. As these consumers carry the quantity stored to the second period, storage is detrimental to the firm and the first period price is distorted upward in order to mitigate consumers' storage incentives. Notably, for larger values of the storage cost, i.e., $c \geq \tilde{c}^u$, the firm cuts the first period price below the static monopoly level to stimulate storage. We know from the previous discussion that this occurs for $\lambda < \tilde{\lambda}$. The firm benefits from storage because a relatively large share of consumers is naïve and thus resorts to the quantity intended for storage in order to satisfy the first period consumption needs. Under linear demand functions (with parallel shift), the first period equilibrium price in the presence of storage is lower than the static monopoly level if and only if the storage cost is above a certain threshold. Remarkably, this threshold can be rather low and thus the firm prefers price cuts to stimulate storage even when it is relatively cheap and the static solution is unfeasible. Proposition 1 also shows that, as storage reduces the second period demand, it follows from the sequential optimality constraint (6) that the price lies below the static monopoly level in the second period.

4.2 Overestimating consumers

When overestimating their preferences, naïve consumers have low utility $u_l(\cdot) \triangleq u_n(\cdot)$ but at the first period purchase stage believe that they share the same high utility $u_h(\cdot) \triangleq u_s(\cdot)$ as sophisticates. Given that naïfs discover low valuation for the good at the first period consumption stage and thus demand falls in the second period, consumers expect that prices decrease over time, i.e., $p_1 > p_2^e$. This removes consumers' incentives to buy in the first period for storage purposes. In other terms, ex ante storage vanishes, i.e., $s(p_1, p_2^e) = 0$. Clearly, no sophisticated consumer stores ex post either, i.e., $\bar{s}_h(p_1, p_2^e) \triangleq \bar{s}_s(p_1, p_2^e) = 0$.

After discovering low valuation for the good at the first period consumption stage, naïfs may choose to leave some quantity for future consumption, i.e., $\bar{s}_l(p_1, p_2^e) \triangleq \bar{s}_n(p_1, p_2^e) > 0$.⁹ Thus, although prices decline over time, ex post storage can emerge. Intuitively, this occurs

⁹Consumers with low valuation might prefer to (partially) return the good as long as this is allowed. In practice, a return policy applies more frequently to durable than to storable goods that are perishable in consumption. In our model, the firm optimally chooses to impose on consumers relatively strict conditions to deter product return. Even when this is unfeasible for the firm (due to regulatory requirements), consumers still face some non-negligible return costs (e.g., paperwork, postage and packing costs as well as refund waiting time) and the firm may adjust prices to further mitigate consumers' incentives for product return. Notably, the main features of dynamic pricing derived in our analysis persist.

if naïfs' actual preferences are sufficiently weaker than what they perceived. After buying the amount of the good that corresponds to the demand for consumption $D_s(p_1)$ driven by their perceived utility $u_s(\cdot)$, naïfs learn their actual preferences and select the consumption level in the first period to maximize their actual utility $u_n(\cdot)$, given the first period price p_1 and the second period expected price p_2^e . The level of ex post storage $\bar{s}_n(p_1, p_2^e)$ per naïve consumer (with low valuation for the good) is determined by the condition for the equalization of intertemporal marginal utilities in (3) (for $k = l$), where $D_1(\cdot) \triangleq D_s(\cdot)$ (described in Section 3) is evaluated at $\theta = 1$ and $s(p_1, p_2^e) = 0$. At the static monopoly prices p_1^{sm} and p_2^m , ex post storage $\bar{s}_n(\cdot)$ decreases with the storage cost c , i.e., $\partial \bar{s}_n(p_1^{sm}, p_2^m) / \partial c = D'_n(p_2^m - c) < 0$. Solving $\bar{s}_n(p_1^{sm}, p_2^m) = D_s(p_1^{sm}) - D_n(p_2^m - c) = 0$ for c yields a threshold $\tilde{c}^o \triangleq p_2^m - D_n^{-1}(D_s(p_1^{sm}))$ such that the static solution is feasible if and only if $c \geq \tilde{c}^o$. The threshold \tilde{c}^o rises with the fraction λ of sophisticates, thereby making the static solution more difficult to achieve. A higher λ inflates the second period demand and the corresponding price, which magnifies consumers' storage incentives. For $c < \tilde{c}^o$, the static solution cannot be implemented, although the static monopoly prices decline over time. As shown in Lemma 4 (see the Appendix), storage arises in equilibrium for relatively small values of the storage cost, in line with the case of underestimating consumers. However, naïfs now engage in storage activities. After finding the good less valuable than at the purchase stage, they keep some leftovers for future consumption.

In light of these results, we can now characterize the equilibrium price dynamics with overestimating consumers. As with underestimating consumers, we focus on the most relevant case where the storage cost is sufficiently low that storage emerges in equilibrium. A full characterization is provided in the proof of Proposition 2.

Proposition 2 *In the first period, the equilibrium price in the presence of storage is higher than the static monopoly price. In the second period, the equilibrium price in the presence of storage is lower than the static monopoly price.*

Proposition 2 shows that the first period price distortions from the static monopoly level substantially differ from the case of underestimating consumers. Storage is definitely detrimental to the firm because it cannot stimulate further consumption in the first period and reduces demand in the second period. The firm mitigates consumers' storage incentives by raising the first period price above the static monopoly level. The result about the second period price in Proposition 2 directly follows from the sequential optimality constraint (6).

4.3 Effects of naïveté on prices

We now explore the impact of consumer naïveté on the firm's equilibrium pricing policy under both directions of naïfs' belief biases. Our results are formalized in the following proposition. We focus on the most relevant case where the storage cost is sufficiently low that storage emerges in equilibrium. A full characterization is provided in the proof of Proposition 3.

Proposition 3 *A. When naïve consumers underestimate their preferences, the equilibrium price in the presence of storage increases with λ in each period if it is sufficiently lower than the static monopoly price in the first period.*

B. When naïve consumers overestimate their preferences, the equilibrium price in the presence of storage decreases with λ in the first period and increases with λ in the second period if λ is high enough.

Point A of Proposition 3 shows that, with naïve underestimating consumers, a larger fraction λ of sophisticates can inflate the equilibrium price with storage in each period. Put differently, instead of magnifying consumers' exploitation and exacerbating the firm's market power, consumer naïveté can induce the firm to charge lower prices, thereby making all consumers better off. To appreciate the rationale for this result, it is helpful to recall that, when underestimating their preferences, naïfs end up consuming the quantity intended for storage. Thus, a larger number of naïfs strengthens the firm's incentives for price cuts to stimulate storage. A sufficient (albeit not necessary) condition for this result is that the equilibrium price is sufficiently lower than the static monopoly price in the first period. In this case, the firm significantly benefits from promoting storage that translates into naïfs' additional consumption. Notably, we find that, under linear demand functions (with parallel shift), the equilibrium price in the presence of storage increases with the number of sophisticates in each period if and only if it is lower than the static monopoly price in the first period. As discussed after Proposition 1, this is the case if and only if the storage cost is above a certain threshold.

Point B of Proposition 3 indicates that, with naïve overestimating consumers, the equilibrium prices with storage move in different directions across periods in response to an increase in the fraction λ of sophisticates, especially when λ is high enough. The first period equilibrium price tends to decrease with λ . As a larger number of sophisticates (with high valuation for the good) inflates the second period demand and the corresponding price, one might expect that the firm should raise the first period price to alleviate naïfs' stronger storage incentives. The rationale for our result is that *ceteris paribus* a larger portion of sophisticates reduces the

impact of naïfs' aggregate storage on the second period demand, which allows the firm to mitigate the price distortion from the static monopoly level. The second period equilibrium price tends to increase with the number of sophisticates because the second period demand becomes higher. To gain further insights, we consider linear demand functions (with parallel shift). When sophisticates' willingness to pay is relatively high, the first period equilibrium price is concave in λ . Thus, for sufficiently small values of λ , a higher λ increases the first period equilibrium price. For any given first period price, a higher λ induces the firm to inflate the second period price because of higher demand. The firm finds it more profitable to sell in the second period and prefers to discourage storage through a higher first period price. As the magnitude of the responsiveness of the second period price to λ rises with sophisticates' willingness to pay, the positive price effect of λ dominates the (previously discussed) negative effect when sophisticates' willingness to pay is relatively high. The second period equilibrium price definitely increases with λ . Hence, when the fraction of naïfs is relatively large (i.e., λ is low enough), the firm reduces the price in each period in response to a larger number of naïfs, which makes all consumers better off, in line with the case of underestimating consumers.

5 Rational consumers

We now consider a setting where all consumers are rational and hold unbiased beliefs about their preferences. As described in Section 3, at the first period purchase stage, consumers attach a probability $\theta \in [0, 1]$ of having high utility $u_h(x_\tau)$ and a complementary probability $1 - \theta$ of having low utility $u_l(x_\tau)$ from the consumption level x_τ . At the consumption stage, consumers learn their actual valuation for the good. We can decompose the analysis into two main cases according to whether consumers buy the good in the first period for storage purposes or not. This crucially depends on the evolution of prices over time. When prices are expected to be higher in the second period, ex ante storage can emerge in equilibrium. We know from (1) that it holds $s(p_1, p_2^e) > 0$ only if prices are linked through the binding storability constraint, i.e., $p_1 + c = p_2$. After learning their valuation for the good, consumers of type $k = h, l$ choose the amount of consumption that satisfies the condition for intertemporal marginal utilities (2). This yields the level of ex post storage $\bar{s}_k(p_1, p_2^e)$ carried to the second period by each consumer of type $k = h, l$. If prices move such that the storability constraint is not binding, i.e., $p_1 + c > p_2$, which occurs (at least) when prices decline over time, ex ante storage vanishes

and only ex post storage can emerge in equilibrium. Notably, the evolution of prices over time is driven by the intertemporal variation of demand. As described in Section 3, the first period demand $D_1(\cdot)$ arises from the consumer expected utility maximization problem. We allow for a general formulation of the second period demand $D_2(\cdot)$, which varies with a parameter ζ , where $\partial D_2/\partial \zeta > 0$. The role of ζ will be clear in the subsequent analysis.

Following the same logic as with naïve underestimating consumers (see Section 4.1), the firm does not have any incentives to reduce the first period price to the extent that even consumers with high valuation store ex post in equilibrium. As these consumers could be induced to store only if ex ante storage is promoted and thus the storability constraint is binding, i.e., $p_1 + c = p_2$, the firm would sell all consumers some units at p_1 instead of p_2 , thereby incurring a cost equal to c . In other terms, the condition for intertemporal marginal utilities (2) is (generally) slack in equilibrium for consumers with high valuation and these consumers do not store ex post, i.e., $\bar{s}_h(p_1, p_2^e) = 0$.¹⁰ Following the same rationale as with naïve overestimating consumers (see Section 4.2), we find that, after learning their preferences, in the first period consumers with low valuation choose the amount of consumption that equalizes their marginal utilities over time and thus the condition for intertemporal marginal utilities (2) is binding in equilibrium. Thus, ex post storage $\bar{s}_l(p_1, p_2^e)$ per consumer with low valuation is driven by the condition for the equalization of intertemporal marginal utilities (3) (for $k = l$).

The firm can set the static monopoly prices p_1^m and p_2^m , where $p_\tau^m \triangleq \arg \max_{p_\tau} p_\tau D_\tau(p_\tau)$, for $\tau \in \{1, 2\}$, if and only if all consumers abstain from storage activities after learning their preferences or, equivalently, the condition for intertemporal marginal utilities (2) holds at the static solution for each consumer of type $k = h, l$. As consumers with low valuation are more eager to store ex post, this condition reduces to $D_1(p_1^m) - D_l(p_2^m - c) \leq 0$. Hence, there exists a threshold $\tilde{c} \triangleq p_2^m - D_l^{-1}(D_1(p_1^m))$ such that the static solution is feasible if and only if $c \geq \tilde{c}$.¹¹

To convey our main results in a compelling manner, we focus on the most relevant case where the storage cost is sufficiently low that storage emerges in equilibrium. As only consumers with low valuation store ex post, it follows from the condition for the equalization

¹⁰When future demand rises substantially, the firm might not be able to prevent ex post storage even by consumers with high valuation. This scenario significantly resembles a classical setting à la Dudine et al. (2006) where all consumers store ex post and storage definitely becomes harmful to the firm. Intuitively, the firm prefers to increase prices to discourage storage. As it will be clear below, this reinforces our results when future demand is high enough. We refer to Lemma 5 (in the Appendix) for technical details.

¹¹This condition is more stringent than in the case where consumers perfectly know their preferences, i.e., $p_1^m + c \geq p_2^m$. Even in the absence of storage incentives at the purchase stage, consumers that discover low valuation for the good may be inclined to store. This complicates the firm's problem and makes static monopoly prices more difficult to implement.

of marginal utilities (3) (for $k = l$) that the difference between the level of aggregate ex ante storage $S(p_1, p_2^e)$ at the first period purchase stage and the level of aggregate ex post storage $\bar{S}(p_1, p_2^e)$ that the firm expects to arise in the second period is given by

$$\Delta S(p_1, p_2^e) \triangleq S(p_1, p_2^e) - \bar{S}(p_1, p_2^e) = \theta S(p_1, p_2^e) - (1 - \theta) [D_1(p_1) - D_l(p_2^e - c)]. \quad (9)$$

If $\Delta S(p_1, p_2^e) > (<) 0$, storage stimulates (depresses) the firm's sales. Promoting ex ante storage can be beneficial to the firm only if it induces additional sales that translate into further consumption. Otherwise, consumers carry this amount to the second period and the firm loses c for each unit sold at p_1 instead of $p_2 = p_1 + c$ (prices are linked through the binding storability constraint). As the expression for $\Delta S(\cdot)$ in (9) reveals, the firm faces a trade-off when promoting ex ante storage. On the one hand, ex ante storage expands the firm's current sales that translate into further consumption by buyers with high valuation for the good (occurring with probability θ) because they consume the entire quantity bought. On the other hand, buyers with low valuation for the good (occurring with probability $1 - \theta$) accumulate in the form of ex post storage not only the entire amount of ex ante storage but also the difference between their expected and actual demand, which depresses the firm's future sales. When the firm removes ex ante storage through prices such that the storability constraint is not binding, i.e., $p_1 + c > p_2$, ex post storage still persists. As ex ante storage stimulates the firm's current sales but ex post storage depresses the firm's future sales, one might think that the firm should promote ex ante storage whenever possible. To gain some insights, it is helpful to note that preventing ex ante storage allows the firm to mitigate consumers' incentives to store ex post. For any first period price, consumers buy a lower quantity compared to the case where ex ante storage is promoted. Furthermore, as prices with only ex post storage are not linked through the binding storability constraint, i.e., $p_1 + c > p_2$, the firm charges a relatively low second period price, which encourages consumption to a further extent.

We characterize the equilibrium storage solution in Lemma 5 (see the Appendix). The firm's choice between promoting ex ante storage and only ex post storage crucially depends on the magnitude of the second period demand $D_2(\cdot)$, as driven by the parameter ζ . When ex ante storage is allowed, a higher second period demand strengthens consumers' incentives to buy for storage purposes in anticipation of a higher second period price. As ex ante storage is gathered by all consumers but ex post storage is only accumulated by consumers with low

valuation (with probability $1 - \theta$), a higher second period demand makes ex ante storage more profitable for the firm. When only ex post storage emerges, a higher second period demand leads to a higher second period price, which merely encourages storage by consumers with low valuation. Promoting only ex post storage becomes less profitable for the firm. Consequently, when the second period demand is relatively high (as implied by a sufficiently high ζ), the firm promotes ex ante storage, which partially translates into ex post storage, and charges prices linked through the binding storability constraint, provided that the storage cost is relatively low. These results share some similarities with those in a classical model à la Dudine et al. (2006), where storage occurs in equilibrium if future demand is sufficiently high. Storage harms the firm because consumers use the quantity stored instead of buying the good at a higher price in the future. Contrary to Dudine et al. (2006), in our framework ex ante storage differs from ex post storage due to time-varying preferences and the promotion of ex ante storage can be beneficial to the firm because it translates into further consumption. As shown below, this crucially affects the equilibrium price dynamics. When the second period demand is not too high (as implied by a sufficiently low ζ), the firm allows only ex post storage by setting prices such that the storability constraint is not binding, as long as storage is relatively cheap.¹² These results significantly differ from Dudine et al. (2006), where storage disappears if future demand does not grow enough. Using linear demands (with parallel shift), we find that there exists a threshold $\widehat{\zeta}$ such that promoting ex ante storage is profit superior if and only if $\zeta > \widehat{\zeta}$. As $\widehat{\zeta} > 0$, this occurs when the second period demand is sufficiently higher than the first period demand.¹³

The results about ex ante storage with rational consumers reflect those with naïve underestimating consumers (see Section 4.1). Such buyers are more inclined to consume after learning high valuation for the good and inflate the second period demand. The firm prefers to implement a pricing policy that leads to ex ante storage in order to stimulate their consumption. In the same vein, the results about only ex post storage with rational consumers correspond to those with naïve overestimating consumers (see Section 4.2). Such buyers are more reluctant to consume after learning low valuation for the good and dampen the second period demand. The firm chooses a pricing policy that allows only ex post storage in order to mitigate con-

¹²When $D_2(\cdot)$ is too high, the second period price p_2 rises to the extent that the condition $p_1 + c > p_2$ is violated and thus the firm cannot allow only ex post storage. Conversely, when $D_2(\cdot)$ is too low, ex ante storage cannot emerge.

¹³We refer to Lemma 5 (in the Appendix) for technical details.

sumers' storage incentives.

In light of these results, we can now characterize the equilibrium price dynamics. We decompose the analysis according to whether ex ante storage or only ex post storage emerges in equilibrium. In the following proposition, we examine the case of ex ante storage.

Proposition 4 *Suppose that the second period demand is sufficiently high so that ex ante storage arises in equilibrium. Then, in the first period, the equilibrium price exhibits the following features: (i) for $c < \tilde{c}$, it is lower (higher) than the static monopoly price if ζ is low (high) enough and θ is high (low) enough, and (ii) for $c \geq \tilde{c}$, it is lower than the static monopoly price. In the second period, it is lower than the static monopoly price.*

The equilibrium price dynamics are more nuanced than in Dudine et al. (2006), where the firm definitely raises the first period price above the static monopoly level to mitigate consumers' storage incentives. Specifically, Proposition 4 indicates that, when the static solution is not feasible, i.e., $c < \tilde{c}$, the first period price distortions in the presence of ex ante storage are driven by the magnitude of demand, as captured by the parameter ζ , and by the probability θ that consumers exhibit high valuation for the good. It follows from the previous discussion that with ex ante storage a lower second period demand $D_2(\cdot)$, as implied by a lower ζ , mitigates the level of ex ante storage because consumers expect a lower second period price. Facing a lower first period demand, the firm prefers to reduce the corresponding price. Furthermore, a rise in the probability θ of high valuation for the good makes it more profitable for the firm to decrease the first period price in order to stimulate ex ante storage. As a result, when ζ is sufficiently low and θ is sufficiently high, the firm prefers to cut the first period price below the static monopoly level. Following the same rationale, a price surge occurs for ζ high enough and θ low enough. When the static solution is feasible, i.e., $c \geq \tilde{c}$, the firm definitely reduces the first period price below the static monopoly level in order to promote ex ante storage. Given that ex ante storage translates into ex post storage by consumers with low valuation, it follows from the sequential optimality constraint (6) that the second period price unambiguously lies below the static monopoly level.

The results in Proposition 4 are significantly related to those in the case of naïve underestimating consumers, where ex ante storage emerges in equilibrium. In particular, we know from Proposition 1 that, for sufficiently small values of the storage cost, if the fraction λ of sophisticates is large enough, the first period price exceeds the static monopoly level. As such

consumers have low valuation for the good, this corresponds to the condition of θ low enough with rational consumers. Given that a lower θ decreases demand also in the first period, the condition of ζ high enough ensures that the amount of ex ante storage is sufficiently large to induce the firm to charge a first period price above the static monopoly level. As with consumer naïveté, we find that, when the storage cost is relatively large to make the static solution feasible, the firm unambiguously reduces the first period price below the static monopoly level in order to promote storage. Differently from consumer naïveté, in a setting with rational consumers and general second period demand, we can also characterize the conditions under which the first period price lies below the static monopoly level even when the static solution is not feasible. The idea is that, with naïve underestimating consumers, the second period demand rises with the fraction $1 - \lambda$ of naïfs (with high valuation for the good), which corresponds to a higher θ with rational consumers. The condition of ζ low enough ensures that the amount of ex ante storage is not too large so that the firm charges a first period price below the static monopoly level.

Now, we turn to the case of only ex post storage.

Proposition 5 *Suppose that the second period demand is not too high so that only ex post storage arises in equilibrium. Then, in the first period, the equilibrium price is higher than the static monopoly price. In the second period, it is lower than the static monopoly price.*

Proposition 5 indicates that, in the presence of only ex post storage, the firm prefers to raise the first period price above the static monopoly level in order to reduce the amount of the good bought in the first period and stored for the second period by consumers with low valuation. As with ex ante storage (which leads to ex post storage as well), we find from the sequential optimality constraint (6) that ex post storage drives the second period price below the static monopoly level. The results in Proposition 5 correspond to those in Proposition 2 for the case of naïve overestimating consumers (with low valuation for the good), where only ex post storage emerges in equilibrium.

6 Robustness and extensions

Our work is robust and can be extended in different directions. We refer to the Supplementary Appendix (Sections 3 through 5) for the formal proofs of our descriptive statements.

Longer time horizon. Our two-period model provides an analytically tractable framework that is rich enough to identify the main effects at play. Furthermore, the depreciation of storable goods implies that they can be usually accumulated only for a limited amount of time. Our analysis can be generalized to a longer time horizon. First, we consider a finite horizon setting (e.g., Dudine et al. 2006; Hendel and Nevo 2013). To make the problem meaningful under consumer naïveté, in each period there exists a share of naïfs with biased beliefs. Consistently with the main analysis (see Sections 4.1 and 4.2), for intermediate values of the storage cost, we find that, with underestimating consumers, the firm reduces prices below the static monopoly level. A longer sequence of periods magnifies the firm’s incentives for price cuts because ex post storage curbs the per period demand. With overestimating consumers, the demand reduction due to ex post storage mitigates the firm’s incentives for price rises above the static monopoly level. Prices are upward distorted in the presence of a sufficiently large fraction of sophisticates so that ex post storage is not too large. Following the same rationale as in the main analysis, similar results hold with rational consumers (see Section 5).

Afterward, we turn to an infinite horizon setting (e.g., Bhaskar and Roketskiy 2024; Hendel et al. 2014; Mittraille and Thille 2016). We consider a plausible framework where consumer preferences are stochastic and follow a (stationary) Markov process. Consistently with the main analysis (see Section 5), we show that, for a sufficiently low storage cost, when promoting ex ante storage, the firm sets prices below (above) the static monopoly level if future demand is not too large (small) and consumers with high (low) valuation are likely enough. With only ex post storage, prices are upward distorted as long as consumers with high valuation are sufficiently likely so that ex post storage is not too large. As in a finite horizon setting, the demand reduction due to ex post storage exacerbates the firm’s incentives for price cuts and alleviates those for price rises.

Nonlinear pricing. Our framework can be extended to nonlinear pricing (e.g., Antoniou and Fiocco 2019; Bhaskar and Roketskiy 2024; Hendel et al. 2014). Intuitively, the firm has incentives to design a contract menu that discriminates across consumers in the second period, after they learned their actual valuation for the good. This generates informational rents for high valuation consumers, namely, naïfs with underestimating consumers and sophisticates with overestimating consumers (see Sections 4.1 and 4.2). Notably, the opportunity to consume the quantity stored provides consumers with some additional surplus in the second period, which leads the firm to offer a discount on the second period payment. The firm intends to recoup

such a discount by charging a higher upfront payment in the first period. Anticipating this, consumers have incentives to skip their purchases in the first period and enjoy the discount in the second period at the cost of no consumption (and no storage) in the first period. We find that, with underestimating consumers, naïfs' reluctance to accept a high upfront payment allows them to retain some surplus, which can be augmented by informational rents in the second period. Naïfs are better off compared to the static solution because they (partially) retain the gains from consuming the quantity stored. Conversely, when overestimating their preferences, naïfs may even end up with losses because they are willing to accept an excessively high upfront payment. The firm exploits naïfs by extracting higher utility from storage than what eventually materializes. This makes naïfs worse off than in the static solution. Following the same rationale as in the main analysis, similar results apply with rational consumers (see Section 5).

Price commitment. It is also interesting to compare the results of our model under limited commitment with those under full commitment, where the firm can credibly announce its future prices. Notably, our results significantly differ from those in Dudine et al. (2006), where storage stems from the firm's opportunistic behavior under limited commitment and the firm charges higher prices than under full commitment. In our framework, in the presence of ex ante storage — as with naïve underestimating consumers (see Section 4.1) or rational consumers and sufficiently high future demand (see Section 5) — the price comparison ultimately hinges upon the price responsiveness of the difference in the firm's gains from storage between full and limited commitment. Prices are higher under full commitment in each period if and only if such difference increases with the price. This condition holds as long as the second period demand is not too convex.

In the presence of only ex post storage — as with naïve overestimating consumers (see Section 4.2) or rational consumers and sufficiently small future demand (see Section 5) — the price comparison varies across periods. Under full commitment, a higher first period price is more effective at reducing storage because limited commitment makes consumers more reluctant to reduce their storage in anticipation of the firm's opportunistic behavior. Thus, for any given second period price, full commitment strengthens the firm's incentives to inflate the first period price. For any given first period price, the opportunity to credibly announce future prices allows the firm to set a lower second period price than under limited commitment to dampen storage. Such insights are corroborated in a framework with linear demands (and

parallel shift), where full commitment leads to a higher (lower) equilibrium price in the first (second) period in comparison with limited commitment.

7 Empirical application

Our work delivers a range of predictions that are amenable to empirical corroboration. As shown in Propositions 4 and 5, the firm may prefer price cuts to promote ex ante storage with increasing demand and price rises to dampen ex post storage with decreasing demand. This indicates that storability may lead to countercyclical pricing, referred to as price decreases (increases) during high (low) demand season. Such intriguing phenomenon has been systematically documented in the empirical literature, which has provided alternative explanations either on the demand side (e.g., Butters et al. 2025; Guler et al. 2014; Haviv 2022; Kwon et al. 2023; MacDonald 2000; Nevo and Hatzitaskos 2006) or on the supply side (e.g., Chevalier et al. 2003; Meza and Sudhir 2006). Endogenizing price dynamics and storage incentives, our model identifies a novel rationale for countercyclical pricing in storable good markets, which is complementary to previous explanations and aims to reconcile the demand-side and supply-side perspectives. Furthermore, as Proposition 4 reveals, our model is rich enough to accommodate for price rises when demand increases significantly. This is consistent with some empirical evidence of procyclical pricing (e.g., Butters et al. 2025; Gijzenberg 2017). Our results also shed new light on the impact of demand uncertainty on prices. It follows from our discussion in Section 5 that, when demand is volatile but constant in expectation (namely, $\zeta = 0$ with linear demands and parallel shift), the firm prefers to allow only ex post storage, which leads to price raises. As a stable and deterministic demand typically yields constant prices over time, our analysis suggests that demand volatility can drive up prices for storable commodities, thereby mitigating or even reversing price countercyclicity. Notably, this does not stem from the standard argument of the firm's attitude toward risk.

To provide a stylized empirical application of our model, we collect data from Camelcamel-camel on Amazon and third party prices in the United States for different storable product categories, namely, vitamin B, vitamin C, multivitamin, soup, tuna, and toilet paper, during the period from January 2010 to June 2024. We find seasonality in demand for all products, using searches through Google Trends. The vitamin market is particularly suitable for different reasons. The Covid-19 pandemic represents an exogenous shock that creates a valuable

opportunity to test our theoretical findings. The demand for vitamin C appears to be more seasonal than the demand for vitamin B and multivitamin. Prices for each vitamin product are countercyclical. Hence, in the subsequent analysis, we focus on the vitamin market.¹⁴ We run the following OLS regression

$$P_{ibt} = \alpha_0 + \alpha_1 Covid + \alpha_2 Vit_c + \alpha_3 High + \alpha_4 Vit_c \times High + \alpha_5 Vit_c \times Covid + \alpha_6 Direct + \alpha_7 Trend_B + v_{ibt}. \quad (10)$$

The dependent variable P_{ibt} denotes the logarithm of the price for vitamin of type i (vitamin B, C or multivitamin) and brand b in period t ; $Covid$ is a dummy variable for Covid-19 that takes value one since March 1, 2020 and zero prior to this date; Vit_c takes value one for vitamin C and zero otherwise; $High$ receives value one for the observations that span during high demand season and zero otherwise;¹⁵ $Direct$ takes value one or zero when the product is sold by Amazon or a third party, respectively; $Trend_B$ controls for a time trend for vitamin B. As illustrated in Figure 1 (at the end of the paper), which plots (monthly) prices for vitamin B, C, and multivitamin, price trends for vitamin C and multivitamin are (almost) parallel from 2013 to March 2020, whereas vitamin B displays an increasing decoupling trend from vitamin C. Hence, a time trend is included for vitamin B. We saturate the regression in (10) with year fixed effects, which control for time-varying effects common to all brands, and with brand fixed effects in order to control for time-invariant brand characteristics. Finally, v_{ibt} is a stochastic disturbance.

A negative (and statistically significant) sign of the coefficient α_3 in the regression in (10) indicates price decreases during high demand season, namely, price countercyclicity. We incorporate into the regression in (10) two interaction terms, α_4 and α_5 , in order to capture additional effects that can be explained in light of our analysis. It follows from Proposition 4 that, with a relatively mild demand increase, the firm may prefer price cuts, whereas a sharp demand increase may induce the firm to raise prices. As demand seasonality for vitamin C tends to be relatively more pronounced than for vitamin B and multivitamin, our results suggest that α_4 is positive. The regression in (10) can be interpreted as a quasi-natural experiment due to

¹⁴We refer to the Supplementary Appendix (Section 6) for details about the empirical application and the investigation of other products.

¹⁵The variable $High$ takes value one during the months where the monthly average search exceeds the average search of the full sample. This corresponds to the period from October to March for vitamin C and multivitamin and to the periods from January to May and from July to October for vitamin B. Notably, high (low) demand season is typically characterized by increasing (decreasing) demand.

Covid-19, which has significantly inflated the demand for vitamins. As previously mentioned, price trends for vitamin C and multivitamin are (almost) parallel from 2013 to March 2020 and a time trend is included for vitamin B. Thus, the interaction term α_5 can be considered as the outcome of a difference-in-differences regression where vitamin C acts as the treated group and multivitamin (and vitamin B) as the control group. Given the huge impact of Covid-19 on the demand for vitamin C, we expect from our model that α_5 is positive.

The results of the regression in (10) are summarized in Table 1 (at the end of the paper). The variables *Vit_c* and *Direct* are dropped due to multicollinearity. The coefficient α_3 is negative and statistically significant (except in the last column) mostly at the 1% level. Price counter-cyclicality can be explained in light of our theoretical findings. Focusing on the full sample, prices tend to be lower between 0.5% and 0.7% during high demand season compared to low demand season. The coefficient α_4 is positive throughout, and statistically significant (at the 5% level) in the first column (which refers to the full sample) and in the fifth column (which refers to vitamin B and C). Hence, during high demand season prices tend to be relatively higher for vitamin C. This provides some corroboration of our presumption that a more pronounced demand rise can lead to higher prices. Price dynamics for vitamin C are also consistent with our model because of higher demand volatility compared to the control group.¹⁶ In line with our prediction, the coefficient α_5 is positive and statistically significant (except in the fifth column) mostly at the 1% level. Including two-way fixed effects that control for unobserved heterogeneity varying at year and brand level, the results are significant at the 1% level throughout. This provides some suggestive evidence on our result that prices become higher when demand rises significantly. As expected, the coefficient α_1 is positive and statistically significant almost always at the 1% level, which captures the impact of a sharp increase in demand for vitamins due to Covid-19.

8 Concluding remarks

In this paper, we investigate a market where a firm sells a storable good to consumers that exhibit time-varying preferences. We provide a general and analytically tractable framework where consumers hold (possibly biased) beliefs about their valuation for the good at the purchase stage and learn their actual valuation at the consumption stage. Time-varying prefer-

¹⁶This is measured by standard deviation in searches (see Table S2 in the Supplementary Appendix).

ences induce consumers to revise their consumption and storage decisions after buying the good. Thus, the amount purchased for storage — i.e., *ex ante* storage — which stimulates the firm's current sales may depart from the quantity actually stored — i.e., *ex post* storage — which depresses the firm's future sales. We show that the firm's dynamic pricing policy and consumers' storage incentives crucially hinge upon on the trade-off that the firm faces when promoting *ex ante* storage. Specifically, *ex ante* storage can be profitable for the firm as long as it spurs further consumption. However, this occurs at the firm's cost of inflating *ex post* storage. The identification of *ex ante* and *ex post* storage crucially relies on a condition for intertemporal marginal utilities from consumption for each buyer. This condition boils down to the storability constraint in a classical setting à la Dudine et al. (2006) where consumer preferences are invariant between the purchase and consumption stage and thus *ex ante* storage coincides with *ex post* storage. With time-varying preferences, we show that the equilibrium price dynamics are more nuanced. The firm may prefer price cuts to promote *ex ante* storage with increasing demand and price rises to dampen *ex post* storage with decreasing demand.

Our work delivers a range of empirically testable predictions about price dynamics in storable good markets. In the stylized empirical application of our model, we provide some corroboration of our theoretical findings. We propose a novel and complementary explanation for the puzzling phenomenon of countercyclical pricing, with the aim of reconciling the well-known demand-side and supply-side perspectives and identifying the role of the magnitude of demand. Notably, our model is rich enough to allow for price procyclicality as well. Our results also shed some light on the impact of demand uncertainty on price levels. An implication of countercyclical pricing that can be empirically investigated is the realization of low markups in growing markets and high markups in shrinking markets. The presence of consumer storage even when prices decline over time can also deserve empirical evaluation.

Our analysis provides potentially significant policy implications, especially in terms of welfare consequences of consumer naïveté. Contrary to what common wisdom suggests, we find that, instead of magnifying consumers' exploitation and exacerbating the firm's market power, consumer naïveté can reduce prices and make consumers better off. This unveils unintended anticompetitive effects of regulatory interventions that are meant to 'educate' consumers and simplify their decision problems by enlarging the scope for mandatory disclosure of different aspects of prices and product characteristics. An accurate assessment of consumers' expectations and biases about their preferences is therefore of paramount importance in markets for

storable goods.

Appendix

This Appendix collects the proofs. When necessary, we introduce intermediate steps and associated proofs. Additional (non-essential) technical details are collected in the Supplementary Appendix (Section 2). To prove Proposition 1, we proceed through the following three steps. In the first step, we provide the main features of the static and dynamic solution.

Lemma 1 *A. The static solution is feasible if and only if $c \geq \tilde{c}^u$. Prices are p_1^{sm} and p_2^m , where $p_1^{sm} + c \geq p_2^m$.*

*B. The dynamic storage solution is feasible if and only if $c < \bar{c}^u$. Prices are p_1^{*u} and $p_2^{*u} = p_1^{*u} + c$, with ex ante storage $S^{*u} > 0$ and ex post storage $\bar{S}^{*u} = \lambda S^{*u} > 0$. For $c \geq \bar{c}^u$, the dynamic solution exhibits no storage. Prices are $p_1^{**u} = p_2^{**u} - c$ and $p_2^{**u} = p_2^m$.*

Proof of Lemma 1. The firm faces three following pricing options: (I) $p_1 + c > p_2$; (II) $p_1 + c = p_2$; (III) $p_1 + c < p_2$.

(I) Suppose $p_1 + c > p_2$. It follows from (1) that $s(p_1, p_2^e) = S(p_1, p_2^e) = 0$. The firm's problem in (5) can be decomposed in two static problems. Differentiating the firm's first and second period profits $p_1 D_s(p_1)$ and $p_2 D_2(p_2)$ (where $D_s(\cdot)$ and $D_2(\cdot)$ are defined in Section 4.1) with respect to p_1 and p_2 yields respectively

$$\phi_1(p_1) \triangleq D_s(p_1) + p_1 D'_s(p_1) \quad \text{and} \quad \phi_2(p_2) \triangleq D_2(p_2) + p_2 D'_2(p_2), \quad (\text{A1})$$

where $\phi'_\tau(\cdot) < 0$, $\tau \in \{1, 2\}$ (by profit concavity). The first and second period static monopoly prices p_1^{sm} and p_2^m , which respectively satisfy $\phi_1(p_1^{sm}) = 0$ and $\phi_2(p_2^m) = 0$, are given by

$$p_1^{sm} = -\frac{D_s(p_1^{sm})}{D'_s(p_1^{sm})} \quad \text{and} \quad p_2^m = -\frac{D_2(p_2^m)}{D'_2(p_2^m)}. \quad (\text{A2})$$

The static solution is feasible if and only if $p_1^{sm} + c \geq p_2^m$ or equivalently $c \geq \tilde{c}^u$, where $\tilde{c}^u \triangleq p_2^m - p_1^{sm}$ (see Section 4.1). Applying the implicit function theorem to $\phi_2(p_2^m) = 0$, where $\phi_2(\cdot)$ is defined by (A1), yields

$$\frac{\partial p_2^m}{\partial \lambda} = \frac{D_n(p_2^m) - D_s(p_2^m) + p_2^m [D'_n(p_2^m) - D'_s(p_2^m)]}{\Pi''_2(p_2^m)}. \quad (\text{A3})$$

By profit concavity, it holds $\partial p_2^m / \partial \lambda < 0$ (the opposite holds with overestimating consumers). As $p_1^{sm} \rightarrow p_2^m$ at $\lambda \rightarrow 1$ and p_1^{sm} does not depend on λ , we find that $\bar{c}^u > 0$ and $\partial \bar{c}^u / \partial \lambda < 0$.

(II) Suppose $p_1 + c = p_2$. It follows from (1) that $s(p_1, p_2^e) = S(p_1, p_2^e) \in [0, D_2(p_1 + c)]$. The firm's problem is given by (5). Proceeding backward and using $p_2 = p_1 + c$, we find from the first-order condition for the firm's second period maximization problem associated with the sequential optimality constraint (6) that $\bar{S}(p_1) = \max\{0, \phi_2(p_1 + c)\}$, where $\phi_2(\cdot)$ is defined by (A1). The following two cases emerge.

(IIa) Let $\bar{S}(p_1) > 0$. This identifies the dynamic storage solution. Sophisticates' ex ante and ex post storage coincide. If naïfs exhaust the good in the first period, we obtain $\bar{S}(p_1) = \phi_2(p_1 + c) = \lambda S(p_1)$. Otherwise, naïfs also carry some quantity to the second period and $\bar{S}(p_1) = \phi_2(p_1 + c) \in (\lambda S(p_1), S(p_1)]$. First, we consider $\bar{S}(p_1) = \phi_2(p_1 + c) = \lambda S(p_1)$, which yields $S(p_1) = \phi_2(p_1 + c) / \lambda$. The firm's maximization problem becomes

$$\max_{p_1} p_1 [D_s(p_1) + S(p_1)] + (p_1 + c) [D_2(p_1 + c) - \lambda S(p_1)]. \quad (\text{A4})$$

Using (A1), the first-order condition for p_1 can be written as

$$\lambda \phi_1(p_1) + \phi_2(p_1 + c) + [p_1(1 - \lambda) - c\lambda] \phi_2'(p_1 + c) = 0. \quad (\text{A5})$$

The equilibrium prices with storage are

$$p_1^{*u} = \frac{\lambda}{1 - \lambda} c - \frac{\lambda \phi_1(p_1^{*u}) + \phi_2(p_1^{*u} + c)}{(1 - \lambda) \phi_2'(p_1^{*u} + c)} \quad \text{and} \quad p_2^{*u} = p_1^{*u} + c. \quad (\text{A6})$$

The equilibrium aggregate ex ante and ex post storage are $S^{*u} \triangleq S(p_1^{*u}) = \phi_2(p_1^{*u} + c) / \lambda$ and $\bar{S}^{*u} \triangleq \bar{S}(p_1^{*u}) = \phi_2(p_1^{*u} + c)$, respectively.

(IIb) For $c \geq \bar{c}^u$, consumers do not store. This identifies the dynamic solution without storage. We find from the sequential optimality constraint (6) that the equilibrium prices are $p_1^{**u} = p_2^{**u} - c$ and $p_2^{**u} = p_2^m$. As the firm's maximization problem in (A4) allows for any $S(\cdot)$, option (IIa) dominates option (IIb) whenever it is feasible, i.e., $\bar{S}^{*u} > 0$.

(III) Suppose $p_1 + c < p_2$. It follows from (1) that $s(p_1) = S(p_1) = D_2(p_1 + c)$. As at least sophisticates carry their entire ex ante storage to the second period, ex post storage cannot be lower than $\lambda D_2(p_1 + c)$. For any p_1 , this level of ex post storage is (weakly) higher than in option (IIa), where $\bar{S}(p_1) = \lambda S(p_1)$ and $S(p_1) \in [0, D_2(p_1 + c)]$. It follows from the

sequential optimality constraint (6) that the firm prefers to set $p_2 \leq p_1 + c$. This contradicts the supposition $p_1 + c < p_2$ and thus option (III) is not implementable.

We refer to the Supplementary Appendix for the derivation of the expression for $s(\cdot)$ in (1), the condition for the feasibility of the dynamic storage solution, the impact of λ on \bar{S}^{*u} , and the case where naïfs carry some quantity to the second period, i.e., $\bar{S}(p_1^{*u}) \in (\lambda S(p_1^{*u}), S(p_1^{*u})]$. ■

In the second step, we establish the condition for the interval $\bar{c}^u < c < \tilde{c}^u$ to be non-empty.

Lemma 2 *There exists a threshold $\tilde{\lambda}$ such that $\bar{c}^u > \tilde{c}^u$ if and only if $\lambda < \tilde{\lambda}$.*

Proof of Lemma 2. Set $c = \tilde{c}^u \triangleq p_2^m - p_1^{sm}$, where the static solution is feasible (see Section 4.1). As $p_1^{sm} + \tilde{c}^u = p_2^m$, substituting $\phi_1(p_1^{sm}) = 0$ and $\phi_2(p_2^m) = 0$, where $\phi_1(\cdot)$ and $\phi_2(\cdot)$ are defined by (A1), into the left-hand side of the first-order condition for p_1^{*u} in (A5) at $c = \tilde{c}^u$ yields $[p_1^{sm}(1 - \lambda) - \tilde{c}^u\lambda]\phi_2'(p_2^m)$. We find from $\phi_2'(\cdot) < 0$ (by profit concavity) that at $c = \tilde{c}^u$ it holds $p_1^{sm} > p_1^{*u}$ if and only if $p_1^{sm}(1 - \lambda) - \tilde{c}^u\lambda > 0$. It holds $\bar{c}^u > \tilde{c}^u$ if and only if $p_1^{sm}(1 - \lambda) - \tilde{c}^u\lambda > 0$. To see this, note that, if $p_1^{sm}(1 - \lambda) - \tilde{c}^u\lambda > 0$, we have $p_1^{sm} > p_1^{*u}$ and thus a price decrease from p_1^{sm} to p_1^{*u} at $c = \tilde{c}^u$ leads from zero storage to positive storage (recall $\partial\bar{S}(\cdot)/\partial p_1 < 0$ from the proof of Lemma 1), which implies $\bar{c}^u > \tilde{c}^u$. Furthermore, if $\bar{c}^u > \tilde{c}^u$, a price change from p_1^{sm} to p_1^{*u} at $c = \tilde{c}^u$ leads from zero storage to positive storage, which implies $p_1^{sm} > p_1^{*u}$ (recall $\partial\bar{S}(\cdot)/\partial p_1 < 0$ from the proof of Lemma 1) and thus $p_1^{sm}(1 - \lambda) - \tilde{c}^u\lambda > 0$. As $\tilde{c}^u \triangleq p_2^m - p_1^{sm}$, there exists a threshold $\tilde{\lambda} \triangleq p_1^{sm}/p_2^m > 0$ such that $p_1^{sm}(1 - \lambda) - \tilde{c}^u\lambda = p_1^{sm} - \lambda p_2^m > 0$ or equivalently $\bar{c}^u > \tilde{c}^u$ if and only if $\lambda < \tilde{\lambda}$.¹⁷

We refer to the Supplementary Appendix for the derivation of the condition for the uniqueness of the threshold $\tilde{\lambda}$. ■

In the final step, we characterize the equilibrium solution.

Lemma 3 A. *Let $\lambda \leq \tilde{\lambda}$. In equilibrium, (i) for $c < \tilde{c}^u$, where $\tilde{c}^u \in [\tilde{c}^u, \bar{c}^u]$, the dynamic storage solution arises, and (ii) for $c \geq \tilde{c}^u$, the static solution arises.*

B. *Let $\lambda > \tilde{\lambda}$. In equilibrium, (i) for $c < \bar{c}^u$, the dynamic storage solution arises, (ii) for $\bar{c}^u \leq c < \tilde{c}^u$, the dynamic solution without storage arises, and (iii) for $c \geq \tilde{c}^u$, the static solution arises.*

Proof of Lemma 3. First, suppose $\lambda \leq \tilde{\lambda}$. It follows from Lemma 2 that $\bar{c}^u \geq \tilde{c}^u$ (where $\bar{c}^u = \tilde{c}^u$ if and only if $\lambda = \tilde{\lambda}$). Let Π^m be the firm's profit in the static solution (derived in point (I) of Lemma 1) and Π^{*u} be the firm's profit in the dynamic storage solution (derived in point (IIa)

¹⁷Note from (A4) that $p_1^{sm}(1 - \lambda) - \tilde{c}^u\lambda > 0$ corresponds to $\partial\Pi(p_1^{sm}, p_1^{sm} + c)/\partial S|_{c=\tilde{c}^u} > 0$.

of Lemma 1). At $c = \tilde{c}^u$ it holds $\Pi^{*u} \geq \Pi^m$ (where $\Pi^{*u} = \Pi^m$ if and only if $\bar{c}^u = \tilde{c}^u$). To see this, note that at $c = \tilde{c}^u$ the firm obtains the static monopoly profit by charging p_1^{sm} and $p_2^m = p_1^{sm} + c$. As $\bar{S}^{*u} > 0$, a revealed preference argument indicates that the firm is better off in the (feasible) dynamic storage solution. At $c = \bar{c}^u$ it holds $\Pi^m \geq \Pi^{*u}$ (where $\Pi^m = \Pi^{*u}$ if and only if $\bar{c}^u = \tilde{c}^u$), because the dynamic solution yields no storage (as shown in point (IIb) of Lemma 1) and the static solution arises from an unconstrained maximization problem. Note that Π^m is independent of c , whereas $\partial\Pi^{*u}/\partial c = [p_1^{*u}(1-\lambda) - c\lambda] \phi_2'(p_1^{*u} + c) / \lambda$. As $\phi_2'(\cdot) < 0$ (by profit concavity), we have $\partial\Pi^{*u}/\partial c < 0$ if and only if $p_1^{*u}(1-\lambda) - c\lambda > 0$. For $c > \tilde{c}^u$, it follows from the static feasibility constraint (7) that $p_2^m < p_1^{sm} + c$. As the sequential optimality constraint (6) implies $p_2^{*u} < p_2^m$, we find that $p_2^{*u} = p_1^{*u} + c < p_1^{sm} + c$, which gives $p_1^{*u} < p_1^{sm}$. Using (A1), we obtain from $p_1^{*u} < p_1^{sm}$ and $p_2^{*u} < p_2^m$ that $\lambda\phi_1(p_1^{*u}) + \phi_2(p_1^{*u} + c) > 0$. As $\phi_2'(\cdot) < 0$ (by profit concavity), it follows from the first-order condition for p_1^{*u} in (A5) that $p_1^{*u}(1-\lambda) - c\lambda > 0$ and thus $\partial\Pi^{*u}/\partial c < 0$. Recalling that at $c = \tilde{c}^u$ it holds $\Pi^{*u} \geq \Pi^m$ and at $c = \bar{c}^u$ it holds $\Pi^m \geq \Pi^{*u}$ (where the equalities hold if and only if $\tilde{c}^u = \bar{c}^u$), we find from the intermediate value theorem that there exists a unique threshold $\hat{c}^u \in [\tilde{c}^u, \bar{c}^u]$ such that $\Pi^{*u} > \Pi^m$ if and only if $c < \hat{c}^u$ (with $\Pi^m = \Pi^{*u}$ if and only if $c = \hat{c}^u$). It follows from Lemma 1 that in equilibrium for $c < \tilde{c}^u$ the dynamic storage solution (derived in point (IIa) of Lemma 1) arises, and for $c \geq \tilde{c}^u$ the static solution (derived in point (I) of Lemma 1) arises. Now, suppose $\lambda > \tilde{\lambda}$. This implies from Lemma 2 that $\tilde{c}^u > \bar{c}^u$. It follows from Lemma 1 that in equilibrium for $c < \bar{c}^u$ the dynamic storage solution arises, for $\bar{c}^u \leq c < \tilde{c}^u$ the dynamic solution without storage (derived in point (IIb) of Lemma 1) arises, and for $c \geq \tilde{c}^u$ the static solution arises. ■

Equipped with the results in Lemmas 1 through 3, we now prove Proposition 1.

Proof of Proposition 1. First, we show price comparisons in the first period. For $c < \tilde{c}^u$, the static feasibility constraint (7) is violated. We know from Lemma 3 that either the dynamic storage solution or the dynamic solution without storage is implemented in equilibrium. First, we consider the dynamic storage solution. Substituting $\phi_1(p_1^{sm}) = 0$, where $\phi_1(\cdot)$ is defined by (A1), into the first-order condition for p_1^{*u} in (A5) yields $[p_1^{sm}(1-\lambda) - c\lambda] \phi_2'(p_1^{sm} + c) + \phi_2(p_1^{sm} + c)$. It follows from $c < \tilde{c}^u$ or equivalently $p_1^{sm} + c < p_2^m$ that $\phi_2(p_1^{sm} + c) > 0$. As $\phi_2'(\cdot) < 0$ (by profit concavity), we find that for $c < \tilde{c}^u$ there exists a unique threshold $\bar{\lambda} \triangleq p_1^{sm} / (p_1^{sm} + c)$ such that a sufficient (albeit not necessary) condition for $p_1^{*u} > p_1^{sm}$ is $\lambda > \bar{\lambda}$. Now, we consider the dynamic solution without storage. Using the results in point (IIb) of the proof of Lemma 1, we obtain from $c < \tilde{c}^u$ or equivalently $p_1^{sm} + c < p_2^m = p_2^{*u}$ that

$p_1^{**u} = p_2^{**u} - c > p_1^{sm}$. For $\hat{c}^u \leq c < \bar{c}^u$, where $\hat{c}^u \in [\bar{c}^u, \bar{c}^u]$, we know from Lemma 3 that the dynamic storage solution emerges in equilibrium. We find that $p_1^{*u} < p_1^{sm}$. To see this, note that, proceeding by contradiction, $p_1^{*u} \geq p_1^{sm}$ yields $p_2^{*u} = p_1^{*u} + c \geq p_1^{sm} + c \geq p_2^m$ (where the last inequality follows from $c \geq \bar{c}^u$). This violates $p_2^{*u} < p_2^m$, as implied by the sequential optimality constraint (6). For $c \geq \max\{\bar{c}^u, \hat{c}^u\}$, it follows from Lemma 3 that the static solution applies in equilibrium. Specifically, if $\lambda \leq \tilde{\lambda}$, the static solution arises in equilibrium for $c \geq \bar{c}^u$, with $\hat{c}^u \in [\bar{c}^u, \bar{c}^u]$, which implies that $\hat{c}^u \geq \bar{c}^u$ or equivalently $\hat{c}^u = \max\{\bar{c}^u, \hat{c}^u\}$. If $\lambda > \tilde{\lambda}$, the static solution emerges in equilibrium whenever it is feasible, i.e., $c \geq \bar{c}^u$, which implies that $\bar{c}^u \geq \hat{c}^u$ or equivalently $\bar{c}^u = \max\{\bar{c}^u, \hat{c}^u\}$. We refer to the Supplementary Appendix for the results with linear demand functions. Now, we turn to price comparisons in the second period. It follows from the sequential optimality constraint (6) that with storage it holds $p_2^{*u} < p_2^m$ and without storage the second period equilibrium price becomes p_2^m . ■

To prove Proposition 2, we first characterize the equilibrium solution.

Lemma 4 *In equilibrium, (i) for $c < \bar{c}^o$, the dynamic storage solution arises, which yields prices p_1^{*o} and p_2^{*o} , with $p_1^{*o} > p_2^{*o}$, as well as ex post storage $\bar{S}^{*o} > 0$, (ii) for $\bar{c}^o \leq c < \bar{c}^o$, the dynamic solution without storage arises, which yields prices p_1^{**o} and $p_2^{*o} = p_2^m$, with $p_1^{**o} > p_2^{*o}$, and (iii) for $c \geq \bar{c}^o$, the static solution arises, which yields prices p_1^{sm} and p_2^m , with $p_1^{sm} > p_2^m$.*

Proof of Lemma 4. The static solution is feasible if and only if $c \geq \bar{c}^o \triangleq p_2^m - D_n^{-1}(D_s(p_1^{sm}))$ (see Section 4.2). As at $p_1^{sm} = \lim_{\lambda \rightarrow 1} p_2^m$ it holds $p_2^m = u'_n(D_n(p_2^m)) > u'_n(D_s(p_1^{sm}))$ (due to $u''_n(\cdot) < 0$ and the assumptions about overestimating consumers), we find that $\bar{c}^o > 0$ for λ high enough. Taking the derivative of \bar{c}^o with respect to λ yields $\partial \bar{c}^o / \partial \lambda = \partial p_2^m / \partial \lambda > 0$ (see (A3) with overestimating consumers). For $c < \bar{c}^o$, the firm's maximization problem is given by (5), where ex post storage $\bar{S}(p_1, p_2^e) = (1 - \lambda) \bar{s}_n(p_1, p_2^e)$ stems from condition (3) (as described in Section 4.2). Proceeding backward, the first-order condition for p_2 associated with the firm's second period maximization problem $\max_{p_2} p_2 [D_2(p_2) - \bar{S}(p_1, p_2^e)]$ writes as

$$D_2(p_2) + p_2 D'_2(p_2) - \bar{S}(p_1, p_2^e) = 0. \quad (\text{A7})$$

As $p_2^e = p_2$, the firm's first period maximization problem becomes

$$\max_{p_1} p_1 D_s(p_1) + p_2 [D_2(p_2) - \bar{S}(p_1, p_2)] \quad s.t. \text{ (3) and (A7)}. \quad (\text{A8})$$

It follows from the envelope theorem that the first-order condition for p_1 is

$$D_s(p_1) + p_1 D'_s(p_1) - p_2 \frac{d\bar{S}(p_1, p_2)}{dp_1} = 0. \quad (\text{A9})$$

Applying the implicit function theorem to (3) and (A7) yields

$$\frac{d\bar{S}(p_1, p_2)}{dp_1} = \frac{(1-\lambda) D'_s(p_1) [2D'_2(p_2) + p_2 D''_2(p_2)]}{2D'_2(p_2) + p_2 D''_2(p_2) + (1-\lambda) D'_n(p_2 - c)} < 0, \quad (\text{A10})$$

where the inequality follows from $\phi'_2(\cdot) < 0$ by profit concavity ($\phi_2(\cdot)$ is defined by (A1)).

Using (A7) and (A9), the equilibrium prices with storage are

$$p_1^{*o} = -\frac{D_s(p_1^{*o}) - p_2^{*o} [d\bar{S}(p_1^{*o}, p_2^{*o})/dp_1]}{D'_s(p_1^{*o})} \quad \text{and} \quad p_2^{*o} = -\frac{D_2(p_2^{*o}) - \bar{S}(p_1^{*o}, p_2^{*o})}{D'_2(p_2^{*o})}. \quad (\text{A11})$$

As $p_1^{*o} > p_1^{sm}$ and $p_2^{*o} < p_2^m$ (see below and the proof of Proposition 2) and $p_1^{sm} > p_2^m$ (see the part immediately after (A3) in the proof of Lemma 1), it holds $p_1^{*o} > p_2^{*o}$. Inserting (A11) into (3), the equilibrium aggregate ex post storage is $\bar{S}^{*o} \triangleq \bar{S}(p_1^{*o}, p_2^{*o}) = (1-\lambda) \bar{s}^{*o} = (1-\lambda) [D_s(p_1^{*o}) - D_n(p_2^{*o} - c)]$.

Substituting $\phi_1(p_1^{sm}) = 0$, with $\phi_1(\cdot)$ defined by (A1), for $c = \bar{c}^o$ (where the static solution is feasible) into the left-hand side of the first-order condition for p_1^{*o} in (A9) yields $-p_2 [d\bar{S}(p_1^{sm}, p_2)/dp_1] > 0$, where the inequality follows from (A10). Hence, we obtain that $p_1^{*o} > p_1^{sm}$, which implies from (A10) that ex post storage should become negative for $c = \bar{c}^o$. As this is not feasible, there exists a threshold $\bar{c}^o < \tilde{c}^o$ such that for $\bar{c}^o \leq c < \tilde{c}^o$ the dynamic storage solution and the static solution are unfeasible and thus the dynamic solution without storage emerges in equilibrium. The first period equilibrium price is such that storage vanishes, i.e., $D_s(p_1) - D_n(p_2 - c) = 0$, and the second period equilibrium price reflects the static monopoly level due to the sequential optimality constraint (6). The equilibrium prices are $p_1^{**o} = D_s^{-1}(D_n(p_2^m - c))$ and $p_2^{**o} = p_2^m$. As $p_1^{**o} > p_1^{sm}$ (see the proof of Proposition 2) and $p_1^{sm} > p_2^m$ (see the part immediately after (A3) in the proof of Lemma 1), it holds $p_1^{**o} > p_2^{**o}$.

We refer to the Supplementary Appendix for the condition for the uniqueness of \bar{c}^o , the impact of λ on \bar{s}^{*o} , and the results with linear demand functions. ■

Proof of Proposition 2. First, we show price comparisons in the first period. For $c < \bar{c}^o$, we know from Lemma 4 that the dynamic storage solution applies in equilibrium. Substituting $\phi_1(p_1^{sm}) = 0$, with $\phi_1(\cdot)$ defined by (A1), into the left-hand side of the first-order condition

for p_1^{*o} in (A9) yields $-p_2 [d\bar{S}(p_1^{sm}, p_2) / dp_1] > 0$, where the inequality follows from (A10). We find from the implicit function theorem that $p_1^{*o} > p_1^{sm}$. For $\bar{c}^o \leq c < \bar{c}^o$, we know from Lemma 4 that the dynamic solution without storage applies in equilibrium. As the second period equilibrium price reflects the static monopoly level due to the sequential optimality constraint (6) but the static solution would induce naïfs' ex post storage and thus cannot be implemented, naïfs' first period marginal utility must be higher than in the static solution in order to deter storage, i.e., $u'_n(D_s(p_1^{*o})) > u'_n(D_s(p_1^{sm}))$. It follows from $u''_n(\cdot) < 0$ and $D'_s(\cdot) < 0$ that $p_1^{*o} > p_1^{sm}$. For $c \geq \bar{c}^o$, we know from Lemma 4 that the static solution applies in equilibrium. Now, we turn to price comparisons in the second period. We find from the sequential optimality constraint (6) that with storage it holds $p_2^{*o} < p_2^m$ and without storage the second period equilibrium price becomes p_2^m . ■

Proof of Proposition 3. We start with the case of underestimating consumers. Taking the derivative of the left-hand side of the first-order condition for p_1^{*u} in (A5) with respect to λ , we obtain after some manipulation $\Omega \triangleq \phi_1(p_1^{*u}) + \partial\phi_2(p_1^{*u} + c) / \partial\lambda - (p_1^{*u} + c)\phi'_2(p_1^{*u} + c) + [p_1^{*u}(1 - \lambda) - c\lambda][\partial\phi'_2(p_1^{*u} + c) / \partial\lambda]$. It follows from the implicit function theorem (and the binding storability constraint) that $\partial p_\tau^{*u} / \partial\lambda > 0$, $\tau \in \{1, 2\}$, if and only if $\Omega > 0$. Let $p_1^{*u} < p_1^{sm}$. This implies from (A1) that $\phi_1(p_1^{*u}) > 0$. Two cases emerge. First, suppose $\partial\phi'_2(p_1^{*u} + c) / \partial\lambda \geq 0$. As $p_1^{*u} + c = p_2^{*u} < p_2^m$ due to the sequential optimality constraint (6), we obtain $\lambda\phi_1(p_1^{*u}) + \phi_2(p_1^{*u} + c) > 0$. This implies from (A5) that $p_1^{*u}(1 - \lambda) - c\lambda > 0$. As $\phi'_2(\cdot) < 0$ (by profit concavity) and $\partial\phi'_2(\cdot) / \partial\lambda \geq 0$ (by supposition), we find from $\partial\phi_2(\cdot) / \partial\lambda < 0$ (with underestimating consumers), where $\phi_2(\cdot)$ is given by (A1), that a sufficient (albeit not necessary) condition for $\Omega > 0$ or equivalently $\partial p_\tau^{*u} / \partial\lambda > 0$, $\tau \in \{1, 2\}$, is $\phi_1(p_1^{*u}) > |\partial\phi_2(p_1^{*u} + c) / \partial\lambda|$. Now, suppose $\partial\phi'_2(p_1^{*u} + c) / \partial\lambda < 0$. We rewrite Ω as $\Omega \triangleq \phi_1(p_1^{*u}) + \partial\phi_2(p_1^{*u} + c) / \partial\lambda - (p_1^{*u} + c)\{\phi'_2(p_1^{*u} + c) + \lambda[\partial\phi'_2(p_1^{*u} + c) / \partial\lambda]\} + p_1^{*u}[\partial\phi'_2(p_1^{*u} + c) / \partial\lambda]$. Given $\phi'_2(\cdot) < 0$ (by profit concavity) and $\partial\phi'_2(\cdot) / \partial\lambda < 0$ (by supposition), the expression in square brackets is negative. As $\partial\phi_2(\cdot) / \partial\lambda < 0$ (with underestimating consumers), where $\phi_2(\cdot)$ is defined by (A1), a sufficient (albeit not necessary) condition for $\Omega > 0$ or equivalently $\partial p_\tau^{*u} / \partial\lambda > 0$, $\tau \in \{1, 2\}$, is $\phi_1(p_1^{*u}) > |\partial\phi_2(p_1^{*u} + c) / \partial\lambda + p_1^{*u}[\partial\phi'_2(p_1^{*u} + c) / \partial\lambda]|$. Thus, we find that $\partial p_\tau^{*u} / \partial\lambda > 0$, $\tau \in \{1, 2\}$, if $\phi_1(p_1^{*u})$ is large enough or, equivalently, if p_1^{*u} is sufficiently lower than p_1^{sm} .

Now, we investigate the case of overestimating consumers. Taking the derivative of the left-hand side of the first-order condition for p_1^{*o} in (A9) with respect to λ and recalling that $\bar{S}(p_1, p_2^e) = (1 - \lambda)\bar{s}_n(p_1, p_2^e)$ (see the proof of Lemma 4), we obtain $p_2^{*o}[d\bar{s}_n(p_1^{*o}, p_2^{*o}) / dp_1] -$

$(1 - \lambda) \{ [\partial p_2(p_1^{*o}) / \partial \lambda] [d\bar{s}_n(p_1^{*o}, p_2^{*o}) / dp_1] + p_2^{*o} [\partial^2 \bar{s}_n(p_1^{*o}, p_2^{*o}) / \partial p_1 \partial \lambda] \}$. Using the implicit function theorem, the sign of $\partial p_1^{*o} / \partial \lambda$ corresponds to the sign of this expression. Given that $d\bar{s}_n(p_1^{*o}, p_2^{*o}) / dp_1 < 0$ (see (A10)) but the sign of the expression in curly brackets is a priori ambiguous, a sufficient (albeit not necessary) condition for $\partial p_1^{*o} / \partial \lambda < 0$ is that λ is high enough. Differentiating the left-hand side of the first-order condition for p_2^{*o} in (A7) with respect to λ and recalling $\bar{S}(p_1, p_2^e) = (1 - \lambda) \bar{s}_n(p_1, p_2^e)$ (see the proof of Lemma 4) yields $D_s(p_2^{*o}) - D_n(p_2^{*o}) + p_2^{*o} [D'_s(p_2^{*o}) - D'_n(p_2^{*o})] + \bar{s}_n(p_1^{*o}, p_2^{*o}) - (1 - \lambda) (d\bar{s}^{*o} / d\lambda)$. By the implicit function theorem, the sign of $\partial p_2^{*o} / \partial \lambda$ corresponds to the sign of this expression. The difference between the first two terms and the expression in square brackets are both positive (with overestimating consumers). We also have $\bar{s}_n(p_1^{*o}, p_2^{*o}) > 0$. As the sign of the last term is a priori ambiguous, a sufficient (albeit not necessary) condition for $\partial p_2^{*o} / \partial \lambda > 0$ is that λ is high enough.

We refer to the Supplementary Appendix for the results with linear demand functions and those about the dynamic solution without storage. ■

To prove Propositions 4 and 5, we first characterize the equilibrium solution.

Lemma 5 *A. When ζ is sufficiently high, ex ante storage $S^* > 0$ and ex post storage $\bar{S}^* > 0$ emerge in equilibrium for $c < \bar{c}^*$. Prices are p_1^* and $p_2^* = p_1^* + c$.*

*B. Otherwise, only ex post storage $\bar{S}^{**} > 0$ emerges in equilibrium for $c < \bar{c}^{**}$. Prices are p_1^{**} and p_2^{**} , with $p_1^{**} + c > p_2^{**}$.*

Proof of Lemma 5. It follows from the sequential optimality constraint (6) that the first-order condition for p_2 is given by

$$D_2(p_2, \zeta) + p_2 D'_2(p_2, \zeta) - \bar{S}(p_1, p_2^e) = 0. \quad (\text{A12})$$

First, suppose that the storability constraint is binding, i.e., $p_1 + c = p_2$. This yields the solution with ex ante storage. Below, we show that this solution emerges in equilibrium for ζ high enough. As $S(p_1, p_2^e) = s(p_1, p_2^e)$ and $\bar{S}(p_1, p_2^e) = (1 - \theta) \bar{s}_l(p_1, p_2^e)$, using (3) (for $k = l$) and (A12) along with $p_1 + c = p_2$ yields $S(p_1) = [D_2(p_1 + c, \zeta) + (p_1 + c) D'_2(p_1 + c, \zeta)] / (1 - \theta) + D_l(p_1) - D_1(p_1)$ and $\bar{S}(p_1) = D_2(p_1 + c, \zeta) + (p_1 + c) D'_2(p_1 + c, \zeta)$. It follows from (5) that the firm's first period maximization problem writes as

$$\max_{p_1} p_1 [D_1(p_1) + S(p_1)] + (p_1 + c) [D_2(p_1 + c, \zeta) - \bar{S}(p_1)] \quad \text{s.t.} \quad (3) \text{ (for } k = l) \text{ and (A12).}$$

By the envelope theorem, the first-order condition for p_1 is given by

$$D_1(p_1) + S(p_1) + p_1 [D'_1(p_1) + S'(p_1)] - (p_1 + c) \bar{S}'(p_1) = 0, \quad (\text{A13})$$

where $S'(p_1)$ and $\bar{S}'(p_1)$ follow from $S(p_1)$ and $\bar{S}(p_1)$ (see above). Using (3) (for $k = l$), we find after some manipulation that the first-order condition for p_1 in (A13) can be rewritten as

$$D_l(p_1) + p_1 D'_l(p_1) + \frac{\bar{S}(p_1)}{1-\theta} + [2D'_2(p_1 + c, \zeta) + (p_1 + c) D''_2(p_1 + c, \zeta)] \frac{\theta p_1 - (1-\theta)c}{1-\theta} = 0.$$

The equilibrium prices with ex ante storage (where the storability constraint is binding) are given by

$$p_1^* = \frac{1-\theta}{\theta} c - \frac{(1-\theta) [D_l(p_1^*) + p_1^* D'_l(p_1^*)] + D_2(p_1^* + c, \zeta) + (p_1^* + c) D'_2(p_1^* + c, \zeta)}{2D'_2(p_1^* + c, \zeta) + (p_1^* + c) D''_2(p_1^* + c, \zeta)} \quad (\text{A14})$$

and $p_2^* = p_1^* + c$. The equilibrium aggregate ex ante and ex post storage are $S^* \triangleq S(p_1^*)$ and $\bar{S}^* \triangleq \bar{S}(p_1^*)$. A sufficient (albeit not necessary) condition for $S^* > 0$, which ensures the feasibility of the solution with ex ante storage (as $S^* < \bar{S}^*$), is that ζ is high enough (as $\partial D_2 / \partial \zeta > 0$). As with consumer naïveté, there exists a threshold \hat{c}^* , where $\hat{c}^* \leq \tilde{c}$, such that for $c < \hat{c}^*$ the solution with ex ante storage arises in equilibrium (the static solution is unfeasible).

Now, suppose that the storability constraint is not binding, i.e., $p_1 + c > p_2$. This yields the solution with only ex post storage. Below, we show that this solution emerges in equilibrium for ζ low enough. As $p_2 = p_2^e$ and $S(\cdot) = 0$, we find from (5) that the firm's first period maximization problem writes as

$$\max_{p_1} p_1 D_1(p_1) + p_2 [D_2(p_2, \zeta) - \bar{S}(p_1, p_2)] \quad \text{s.t. (3) (for } k = l \text{) and (A12).}$$

By the envelope theorem, the first-order condition for p_1 is given by

$$D_1(p_1) + p_1 D'_1(p_1) - p_2 \frac{d\bar{S}(p_1, p_2)}{dp_1} = 0. \quad (\text{A15})$$

Using (A12) and (A15), we obtain that the equilibrium prices with only ex post storage (where the storability constraint is not binding) are given by

$$p_1^{**} = -\frac{D_1(p_1^{**}) - p_2^{**} [d\bar{S}(p_1^{**}, p_2^{**}) / dp_1]}{D'_1(p_1^{**})} \quad \text{and} \quad p_2^{**} = -\frac{D_2(p_2^{**}, \zeta) - \bar{S}(p_1^{**}, p_2^{**})}{D'_2(p_2^{**}, \zeta)}. \quad (\text{A16})$$

It follows from the condition for the equalization of intertemporal marginal utilities (3) (for $k = l$), where $s(\cdot) = 0$ (recall $\bar{S}(p_1, p_2^e) = (1 - \theta) \bar{s}_l(p_1, p_2^e)$), that the equilibrium aggregate ex post storage is $\bar{S}^{**} \triangleq \bar{S}(p_1^{**}, p_2^{**}) = (1 - \theta) [D_1(p_1^{**}) - D_l(p_2^{**} - c)]$. As with consumer naïveté, there exists a threshold \hat{c}^{**} , where $\hat{c}^{**} \leq \tilde{c}$, such that for $c < \hat{c}^{**}$ the solution with only ex post storage arises in equilibrium (the static solution is unfeasible).

Now, we compare the two storage solutions. In the solution with ex ante storage, we find from the sequential optimality constraint (6) that, for any p_1 and p_2 (where $p_2 = p_1 + c$), a rise in $D_2(\cdot)$, driven by a higher ζ (as $\partial D_2 / \partial \zeta > 0$), requires an increase in $\bar{S}(\cdot)$, which emerges with probability $1 - \theta$. It follows from the condition for the equalization of intertemporal marginal utilities (3) that each additional unit of $\bar{S}(\cdot)$ allows for an additional unit of $S(\cdot)$. As this is bought by all consumers, the solution with ex ante storage becomes more profitable for the firm. Conversely, in the solution with only ex post storage, we find from the sequential optimality constraint (6) that, for any p_1 , a rise in $D_2(\cdot)$ leads to a higher p_2 . The condition for the equalization of intertemporal marginal utilities (3) implies that $\bar{S}(\cdot)$ increases, which makes this solution less profitable. Consequently, when $D_2(\cdot)$ is sufficiently high, as driven by ζ high enough, the solution with ex ante storage emerges in equilibrium for $c < \hat{c}^*$. Otherwise, the solution with only ex post storage emerges in equilibrium for $c < \hat{c}^{**}$. Note that, when $D_2(\cdot)$ is too high, the second period price p_2 is also significantly high, which violates the storability constraint $p_1 + c > p_2$ and makes the solution with only ex post storage unfeasible. When $D_2(\cdot)$ is too low, as driven by ζ low enough, the solution with ex ante storage cannot be implemented, because the expression for ex ante storage would be negative.

We refer to the Supplementary Appendix for the case where consumers with high valuation also have incentives to store ex post and the results with linear demand functions. ■

Proof of Proposition 4. For $c < \tilde{c}$, evaluating the left-hand side of the first-order condition for p_1^* in (A13) at p_1^m and using $\bar{S}(\cdot)$ (see the proof of Lemma 5), we obtain, after multiplying by $1 - \theta$, the following expression

$$(1 - \theta) [D_l(p_1^m) + p_1^m D_l'(p_1^m)] + D_2(p_1^m + c, \zeta) + (p_1^m + c) D_2'(p_1^m + c, \zeta) \\ + [2D_2'(p_1^m + c, \zeta) + (p_1^m + c) D_2''(p_1^m + c, \zeta)] [\theta p_1^m - (1 - \theta) c].$$

It holds $p_1^* < (>) p_1^m$ when this expression is negative (positive). As the expression in the first square brackets in the second line is negative (by profit concavity), we find that the expression

in the second line is negative if θ is high enough. Furthermore, for ζ low enough, we have $p_2^m < p_1^m + c$, which implies that the sum of the last two terms in the first line is negative. As for θ high enough the first term in the first line is relatively small, we find that, for ζ low enough and θ high enough, the entire expression is negative and thus $p_1^* < p_1^m$.¹⁸ Conversely, if θ is low enough, the expression in the second line is positive. Furthermore, for ζ high enough, we have $p_2^m > p_1^m + c$, which implies that the sum of the last two terms in the first line is positive. As there exist sufficiently large values of ζ such that the expression in the first line is positive, we find that, for ζ high enough and θ low enough, the entire expression is positive and thus $p_1^* > p_1^m$. Turning to the case where $c \geq \tilde{c}$, we show that $p_1^* < p_1^m$. It follows from the sequential optimality constraint (6) that $p_2^* = p_1^* + c < p_2^m$. To show $p_1^* < p_1^m$, it suffices to demonstrate that $p_2^m < p_1^m + c$. Proceeding by contradiction, suppose that $p_2^m \geq p_1^m + c$ or equivalently $c \leq p_2^m - p_1^m$. As $\tilde{c} \triangleq p_2^m - D_l^{-1}(D_1(p_1^m))$ (see Section 5), we have $c \geq \tilde{c}$ if and only if $c \geq p_2^m - D_l^{-1}(D_1(p_1^m))$. It follows from $D_1(p_1) > D_l(p_1)$ that $p_2^m - D_l^{-1}(D_1(p_1^m)) > p_2^m - p_1^m$. This implies that for $c \geq \tilde{c}$ we cannot have $c \leq p_2^m - p_1^m$. Hence, for $c \geq \tilde{c}$ it holds $p_1^* < p_1^m$. The sequential optimality constraint (6) yields $p_2^* < p_2^m$.

We refer to the Supplementary Appendix for the results with linear demand functions. ■

Proof of Proposition 5. Evaluating the left-hand side of the first-order condition for p_1^{**} in (A15) at p_1^m yields $-p_2 [d\bar{S}(p_1^m, p_2) / dp_1] > 0$, where the inequality holds if and only if $d\bar{S}(p_1^m, p_2) / dp_1 < 0$. Applying the implicit function theorem to the condition for the equalization of intertemporal marginal utilities (3) (for $k = l$), where $s(\cdot) = 0$ (recall $\bar{S}(p_1, p_2^e) = (1 - \theta) \bar{s}_l(p_1, p_2^e)$), and (A12) yields

$$\frac{d\bar{S}(p_1, p_2)}{dp_1} = \frac{(1 - \theta) D_1'(p_1) [2D_2'(p_2, \zeta) + p_2 D_2''(p_2, \zeta)]}{2D_2'(p_2, \zeta) + p_2 D_2''(p_2, \zeta) + (1 - \theta) D_1'(p_2 - c)} < 0,$$

where the inequality follows from profit concavity. Then, it holds $p_1^{**} > p_1^m$. Using the sequential optimality constraint (6), we find that $p_2^{**} < p_2^m$. ■

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¹⁸A sufficient (albeit not necessary) condition for the expression in square brackets in the first line to be negative is $D_h'(\cdot) \geq D_l'(\cdot)$, which reinforces this result.

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Figure 1: Price trends for vitamin B, vitamin C, and multivitamin.

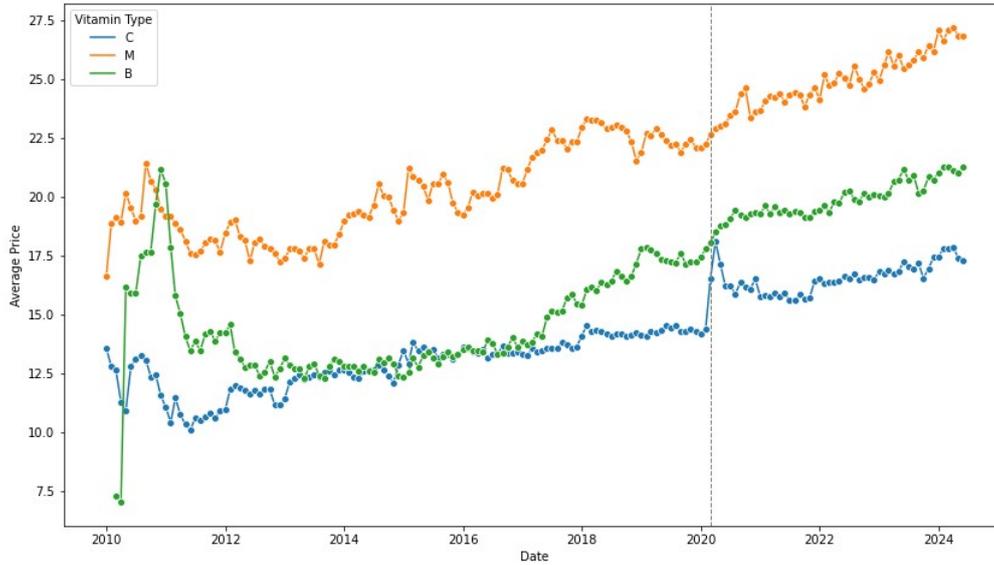


Table 1: OLS regressions for vitamins.

The table reports coefficient estimates and t-statistics (in parentheses) from estimations for vitamin B, vitamin C, and multivitamin. The sample period is January 2010-June 2024. The dependent variable is the natural logarithm of price. Estimation method is OLS with the fixed effects reported in the lower part of the table and robust standard errors clustered by brand. Columns (1) and (2) refer to the whole sample, columns (3) and (4) exclude vitamin B, columns (5) and (6) exclude multivitamin. The lower part of the table also reports the number of observations, the number of brands, and the adjusted R-squared. The ***, **, and * marks denote statistical significance at the 1%, 5%, and 10% level, respectively.

	(1)	(2)	(3)	(4)	(5)	(6)
Covid	0.049*** (5.37)	0.040*** (5.48)	0.059*** (4.66)	0.049*** (5.60)	0.049** (2.48)	0.029** (2.37)
High	-0.007*** (-3.87)	-0.005*** (-2.86)	-0.005** (-2.24)	-0.006*** (-2.70)	-0.010*** (-2.70)	-0.003 (-1.44)
Vit_c×High	0.009** (2.01)	0.006 (1.43)	0.006 (1.56)	0.006 (1.65)	0.012** (2.09)	0.004 (1.07)
Vit_c×Covid	0.048*** (2.13)	0.101*** (5.04)	0.043* (1.77)	0.092*** (4.46)	0.057 (1.55)	0.112*** (5.053)
Trend_B	0.018*** (3.89)	0.032*** (3.47)			0.019** (2.12)	0.034*** (3.71)
Constant	-9.783*** (3.06)	-18.87*** (-3.04)	2.705*** (81.07)	2.753*** (691.70)	-18.609* (-1.88)	-35.49*** (-3.46)
Observations	51,038	50,975	33,774	33,713	31,263	31,191
No of Brands	482	482	317	317	297	297
Brand Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Brand × Year FE	No	Yes	No	Yes	No	Yes
Clustering	Brand	Brand	Brand	Brand	Brand	Brand
Adj-R2	0.86	0.94	0.87	0.95	0.82	0.92

Time-varying preferences for storable goods

Supplementary Appendix

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October 2025

1 Introduction

This Supplementary Appendix complements the paper and proceeds as follows. Section 2 provides additional (non-essential) technical details of the proofs in the Appendix of the paper. Section 3 extends the analysis to a longer time horizon both in a finite and an infinite horizon setting. Section 4 explores the case of nonlinear pricing. Section 5 examines the role of the firm's commitment to future prices. Section 6 describes further aspects of the empirical application of our model.

2 Additional proofs

We provide additional (non-essential) technical details of the proofs in the Appendix of the paper.

Proof of Lemma 1. We derive the expression for ex ante storage $s(p_1, p_2^e)$ in equation (1) of the paper. Given any price sequence $\{p_\tau\}_{\tau=1}^2$, consumers purchase q_τ and then consume x_τ in each period $\tau \in \{1, 2\}$. They also choose the storage level z in the first period. At the first period purchase stage, all consumers maximize their (expected) aggregate net utility $\Psi(x_\tau, z) \triangleq U(x_\tau, y_\tau) - q_1 p_1 - q_2 p_2^e - cz$, where $U(x_\tau, y_\tau)$ is the (expected) aggregate gross utility from the consumption level x_τ and money y_τ in period $\tau \in \{1, 2\}$. Differentiating $\Psi(\cdot)$ with respect to z subject to $q_1 = x_1 + z$ and $q_2 = x_2 - z$ yields $\partial\Psi/\partial z = -p_1 - c + p_2^e$. Then, if $p_1 + c < p_2^e$ (i.e., $\partial\Psi/\partial z > 0$), consumers prefer to buy the amount of the good that allows storing the entire quantity consumed in the second period.¹ If $p_1 + c = p_2^e$ (i.e., $\partial\Psi/\partial z = 0$), they are indifferent between buying for storage purposes and waiting until the second period. If $p_1 + c > p_2^e$ (i.e., $\partial\Psi/\partial z < 0$), they prefer to buy only for current consumption. Ex ante

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¹In anticipation that consumer actual preferences will be realized before the second period starts, for $p_1 + c < p_2^e$ the level of ex ante storage corresponds to the entire second period demand $D_2(p_1 + c)$, provided that arbitrage is allowed. Otherwise, under consumer naïveté, all consumers intend to store $D_s(p_1 + c)$. This can be incorporated into our model without affecting the qualitative results.

storage $s(p_1, p_2^e)$ in equation (1) of the paper reflects the storage level z that maximizes $\Psi(\cdot)$ subject to the associated constraints.

Now, we characterize the condition for the feasibility of the dynamic storage solution, i.e., $\bar{S}^{*u} > 0$. Differentiating \bar{S}^{*u} with respect to c yields $\partial \bar{S}^{*u} / \partial c = (\partial p_2^{*u} / \partial c) \phi_2'(p_2^{*u}) < 0$, where the inequality holds if and only if $\partial p_2^{*u} / \partial c > 0$ (as $\phi_2'(\cdot) < 0$ by profit concavity). To show $\partial p_2^{*u} / \partial c > 0$, we compute the derivative of the left-hand side of the first-order condition for p_2^{*u} — obtained by replacing p_1 with $p_2 - c$ in equation (A5) of the paper — with respect to c . This yields after some manipulation $-\lambda \phi_1'(p_2^{*u} - c) - \phi_2'(p_2^{*u}) > 0$, where the inequality follows from $\phi_\tau'(\cdot) < 0$, $\tau \in \{1, 2\}$ (by profit concavity). We obtain from the implicit function theorem that $\partial p_2^{*u} / \partial c > 0$. Now, we demonstrate that $\bar{S}^{*u} > 0$ at $c = 0$. This corresponds to $D_2(p_1^{*u}) + p_1^{*u} D_2'(p_1^{*u}) > 0$. Using expressions in (A1) of the paper, this is equivalent to $p_1^{*u} = p_2^{*u} < p_2^m$. Substituting $\phi_2(p_2^m) = 0$ into the left-hand side of the first-order condition for p_1^{*u} in equation (A5) of the paper evaluated at $c = 0$ yields $\lambda \phi_1(p_2^m) + p_2^m (1 - \lambda) \phi_2'(p_2^m) < 0$, where the inequality follows from $\phi_1(p_2^m) < 0$ (due to $p_2^m > p_1^{sm}$, as implied by the part immediately after expression in (A3) of the paper) and $\phi_2'(\cdot) < 0$ (by profit concavity). This implies $p_1^{*u} = p_2^{*u} < p_2^m$ and thus $\bar{S}^{*u} > 0$ at $c = 0$. As $\bar{S}^{*u} < 0$ for c arbitrarily large, we find from $\partial \bar{S}^{*u} / \partial c < 0$ (see above) that there exists a unique threshold $\bar{c}^u > 0$ such that $\bar{S}^{*u} > 0$ if and only if $c < \bar{c}^u$.

Now, we derive the condition ensuring that \bar{S}^{*u} decreases with λ , which implies that \bar{c}^u decreases with λ as well. Taking the derivative of $\bar{S}^{*u} = \bar{S}(p_1^{*u}(\lambda), \lambda)$ with respect to λ yields $d\bar{S}^{*u} / d\lambda = [\partial \bar{S}(p_1^{*u}) / \partial p_1] (\partial p_1^{*u} / \partial \lambda) + \partial \bar{S}^{*u} / \partial \lambda$. As \bar{S}^{*u} is driven by the first-order condition for p_2 , which stems from the sequential optimality constraint (6) of the paper, we obtain that $\partial \bar{S}^{*u} / \partial \lambda = D_s(p_2^{*u}) - D_n(p_2^{*u}) + p_2^{*u} [D_s'(p_2^{*u}) - D_n'(p_2^{*u})] < 0$, where the inequality follows from the assumptions about underestimating consumers, and $\partial \bar{S}(p_1^{*u}) / \partial p_1 = 2D_2'(p_1^{*u} + c) + (p_1^{*u} + c) D_2''(p_1^{*u} + c) < 0$, where the inequality follows from profit concavity. Thus, a sufficient (albeit not necessary) condition for $d\bar{S}^{*u} / d\lambda < 0$ is $\partial p_1^{*u} / \partial \lambda \geq 0$. It follows from $d\bar{S}^{*u} / d\lambda < 0$ that $\partial \bar{c}^u / \partial \lambda < 0$. Using linear demand functions $D_s(p_\tau) = \alpha_s - \beta p_\tau$ and $D_n(p_\tau) = \alpha_n - \beta p_\tau$, where $\alpha_n > \alpha_s$, we find from the expression in (A6) of the paper that $d\bar{S}^{*u} / d\lambda < 0$ and $\partial \bar{c}^u / \partial \lambda < 0$.

Finally, suppose that at the prices in (A6) of the paper naïfs carry some quantity to the second period, i.e., $\bar{S}(p_1^{*u}) \in (\lambda S(p_1^{*u}), S(p_1^{*u})]$. When sophisticates and naïfs have sufficiently similar preferences, naïfs may also have incentives to store ex post. This occurs if and only if naïfs' marginal utility from consumption (inflated by the storage cost) is lower in the first period, i.e., $u_n'(D_s(p_1^{*u}) + s(p_1^{*u})) + c < u_n'(D_n(p_2^{*u})) = p_2^{*u}$ or equivalently $u_n'(D_s(p_1^{*u}) + s(p_1^{*u})) < p_1^{*u}$ (recall $p_2^{*u} = p_1^{*u} + c$). It follows from the firm's objective function in (5) of the paper that each unit of ex post storage would impose a cost c on the firm. Then, if at the prices in (A6) of the paper naïfs store ex post, the firm prefers to set higher prices that mitigate the distortion from the benchmark case of perfectly aware consumers (recall $\partial \bar{S}(\cdot) / \partial p_1 < 0$) up to the level where only sophisticates store ex post. The equilibrium prices become \tilde{p}_1^{*u} and $\tilde{p}_2^{*u} = \tilde{p}_1^{*u} + c$, with $\tilde{p}_\tau^{*u} > p_\tau^{*u}$, $\tau \in \{1, 2\}$, where \tilde{p}_1^{*u} is such that only sophisticates store ex post, i.e., $\bar{S}(\tilde{p}_1^{*u}) = \phi_2(\tilde{p}_1^{*u} + c) = \lambda S(\tilde{p}_1^{*u})$, and naïfs' marginal utilities are equalized over time, i.e., $u_n'(D_n(\tilde{p}_1^{*u}) + s(\tilde{p}_1^{*u})) = \tilde{p}_1^{*u}$. With linear demands

(see above), there exists a unique threshold $\tilde{\alpha}_s$ such that for $\alpha_s \leq \tilde{\alpha}_s$ the prices p_1^{*u} and p_2^{*u} emerge in equilibrium. For $\alpha_s > \tilde{\alpha}_s$, the equilibrium prices become \tilde{p}_1^{*u} and \tilde{p}_2^{*u} . To complete the proof, we show that naïfs' ex post storage cannot arise in equilibrium. Proceeding by contradiction, it follows from the sequential optimality constraint (6) of the paper, where $\bar{s}(p_1)$ is replaced by sophisticates' and naïfs' ex post storage $\bar{s}_s(p_1)$ and $\bar{s}_n(p_1)$, weighted by λ and $1 - \lambda$ respectively, that the first-order condition for the firm's second period maximization problem is $D_2(p_2) - \lambda \bar{s}_s(p_1) - (1 - \lambda) \bar{s}_n(p_1) + p_2 D_2'(p_2) = 0$. Note that $\bar{s}_n(p_1)$ is such that naïfs' marginal utilities are equalized over time, i.e., $u_n'(D_s(p_1) + \bar{s}_s(p_1) - \bar{s}_n(p_1)) + c = u_n'(D_n(p_2)) = p_2$, where the quantity intended for storage at the purchase stage $\bar{s}_s(p_1)$ corresponds to sophisticates' ex post storage (because all consumers behave identically at the purchase stage and sophisticates do not revise their storage decisions). This implies that $\bar{s}_n(p_1) = D_s(p_1) + \bar{s}_s(p_1) - D_n(p_1)$ (recall $p_2 = p_1 + c$). Using the first-order condition for the firm's second period maximization problem, we find after some manipulation that $\bar{s}_s(p_1) = D_2(p_1 + c) + (p_1 + c) D_2'(p_1 + c) + (1 - \lambda) [D_n(p_1) - D_s(p_1)]$ and $\bar{s}_n(p_1) = D_2(p_1 + c) + (p_1 + c) D_2'(p_1 + c) - \lambda [D_n(p_1) - D_s(p_1)]$. The firm's profit maximization problem can be written as

$$\max_{p_1} p_1 [D_s(p_1) + \bar{s}_s(p_1)] + (p_1 + c) [D_2(p_1 + c) - \lambda \bar{s}_s(p_1) - (1 - \lambda) \bar{s}_n(p_1)].$$

Using the expressions for $\bar{s}_s'(p_1)$ and $\bar{s}_n'(p_1)$, along with $\bar{s}_n(p_1) = D_s(p_1) + \bar{s}_s(p_1) - D_n(p_1)$, we find from the envelope theorem that the first-order condition for p_1 becomes after some manipulation

$$\bar{s}_n(p_1) + D_n(p_1) + p_1 [\lambda D_s'(p_1) + (1 - \lambda) D_n'(p_1)] - c [2D_2'(p_1 + c) + (p_1 + c) D_2''(p_1 + c)] = 0.$$

Substituting this condition into $\phi_2(\cdot)$, where $\phi_2(\cdot)$ is defined by (A1) of the paper, we obtain $-\bar{s}_n(p_1) - \lambda [D_n(p_1) - D_s(p_1)] + c [2D_2'(p_1 + c) + (p_1 + c) D_2''(p_1 + c)]$. As the expressions in the first and second square brackets are respectively positive (with underestimating consumers) and negative (by profit concavity), it holds $p_2 = p_1 + c \geq p_1 > p_2^m$ for $\bar{s}_n(p_1) > 0$. This violates the sequential optimality constraint (6) of the paper and thus naïfs' ex post storage cannot arise in equilibrium. ■

Proof of Lemma 2. We derive the condition for the uniqueness of the threshold $\tilde{\lambda}$. Let $Y(\lambda) \triangleq p_1^{sm} - \lambda p_2^m$. As $Y(\lambda) \rightarrow p_1^{sm} > 0$ for $\lambda \rightarrow 0$ and $Y(\lambda) \rightarrow 0$ for $\lambda \rightarrow 1$, a sufficient (albeit not necessary) condition for a unique $\tilde{\lambda} > 0$ such that $Y(\lambda) > 0$ if and only if $\lambda < \tilde{\lambda}$ is that $Y(\lambda)$ is (weakly) convex in λ , i.e., $\partial^2 Y / \partial \lambda^2 = -2(\partial p_2^m / \partial \lambda) - \lambda (\partial^2 p_2^m / \partial \lambda^2) \geq 0$. It follows from the expression in (A3) of the paper that

$$\begin{aligned} \frac{\partial^2 p_2^m}{\partial \lambda^2} &= \frac{\partial p_2^m}{\partial \lambda} \frac{2[D_n'(p_2^m) - D_s'(p_2^m)] + p_2^m [D_n''(p_2^m) - D_s''(p_2^m)]}{\Pi_2''(p_2^m)} \\ &+ \frac{D_n(p_2^m) - D_s(p_2^m) + p_2^m [D_n'(p_2^m) - D_s'(p_2^m)]}{[\Pi_2''(p_2^m)]^2} \\ &\times \left\{ 2[D_n'(p_2^m) - D_s'(p_2^m)] + p_2^m [D_n''(p_2^m) - D_s''(p_2^m)] - \frac{\partial p_2^m}{\partial \lambda} \Pi_2'''(p_2^m) \right\}. \end{aligned}$$

Then, it holds $\partial^2 Y / \partial \lambda^2 \geq 0$ if and only if

$$\begin{aligned} & \lambda \frac{\partial p_2^m}{\partial \lambda} \left\{ 2 [D'_n(p_2^m) - D'_s(p_2^m)] + p_2^m [D''_n(p_2^m) - D''_s(p_2^m)] \right\} \Pi_2''(p_2^m) \\ & + 2 [\Pi_2''(p_2^m)]^2 \frac{\partial p_2^m}{\partial \lambda} + \lambda \left\{ D_n(p_2^m) - D_s(p_2^m) + p_2^m [D'_n(p_2^m) - D'_s(p_2^m)] \right\} \\ & \times \left\{ 2 [D'_n(p_2^m) - D'_s(p_2^m)] + p_2^m [D''_n(p_2^m) - D''_s(p_2^m)] - \frac{\partial p_2^m}{\partial \lambda} \Pi_2'''(p_2^m) \right\} \leq 0. \end{aligned}$$

Using (A3) of the paper, we obtain after some manipulation that it holds $\partial^2 Y / \partial \lambda^2 \geq 0$ if and only if $2 [2D'_n(p_2^m) + p_2^m D''_n(p_2^m)] - \lambda (\partial p_2^m / \partial \lambda) \Pi_2'''(p_2^m) \leq 0$. As the expression in square brackets is negative (by profit concavity) and $\partial p_2^m / \partial \lambda < 0$ (see (A3) of the paper), we find that a sufficient (albeit not necessary) condition for $\partial^2 Y / \partial \lambda^2 \geq 0$ is $\Pi_2'''(p_2^m) \leq 0$. Thus, if $\Pi_2'''(p_2^m) \leq 0$, there exists a unique threshold $\tilde{\lambda}$ such that $\bar{c}^u > \bar{c}^m$ if and only if $\lambda < \tilde{\lambda}$. Using linear demand functions $D_s(p_\tau) = \alpha_s - \beta p_\tau$ and $D_n(p_\tau) = \alpha_n - \beta p_\tau$, where $\alpha_n > \alpha_s$, we find from the expressions in (A1) of the paper that $Y(\lambda) > 0$ if and only if $\lambda < \tilde{\lambda}$, where $\tilde{\lambda} = \alpha_s / (\alpha_n - \alpha_s) > 0$ (it holds $\tilde{\lambda} < 1$ if and only if $\alpha_n > 2\alpha_s$). ■

Proof of Proposition 1. Using linear demand functions of the form $D_s(p_\tau) = \alpha_s - \beta p_\tau$ and $D_n(p_\tau) = \alpha_n - \beta p_\tau$, where $\alpha_n > \alpha_s$, we obtain that $p_1^{*u} < p_1^{*m}$ if and only if $c > (\alpha_n - 2\alpha_s) / (2\beta)$. ■

Proof of Lemma 4. We identify the conditions for the uniqueness of the threshold \bar{c}^o below which the dynamic storage solution applies. Differentiating $\bar{S}^{*o} = \bar{S}(p_1^{*o}(c), c)$ with respect to c yields

$$\frac{d\bar{S}^{*o}}{dc} = \frac{d\bar{S}(p_1^{*o}, p_2^{*o})}{dp_1} \frac{\partial p_1^{*o}}{\partial c} + \frac{\partial \bar{S}^{*o}}{\partial c}.$$

Applying the implicit function theorem to the expressions in (3) and (A7) of the paper, we obtain that

$$\frac{\partial \bar{S}^{*o}}{\partial c} = \frac{(1 - \lambda) D'_n(p_1^{*o}) [2D'_2(p_2^{*o}) + p_2^{*o} D''_2(p_2^{*o})]}{2D'_2(p_2^{*o}) + p_2^{*o} D''_2(p_2^{*o}) + (1 - \lambda) D'_n(p_2^{*o} - c)} < 0,$$

where the inequality follows from $\phi'_2(\cdot) < 0$ by profit concavity ($\phi_2(\cdot)$ is defined by (A1) of the paper). As $d\bar{S}(p_1^{*o}, p_2^{*o}) / dp_1 < 0$ (see (A10) of the paper), a sufficient (albeit not necessary) condition for $d\bar{S}^{*o} / dc < 0$ is $\partial p_1^{*o} / \partial c > 0$. Taking the derivative of the left-hand side of the first-order condition for p_1^{*o} in (A9) of the paper with respect to c yields $-\partial p_2(p_1^{*o}) / \partial c [d\bar{S}(p_1^{*o}, p_2^{*o}) / dp_1] - p_2^{*o} [\partial^2 \bar{S}(p_1^{*o}, p_2^{*o}) / \partial p_1 \partial c]$. Applying the implicit function theorem to the expressions in (3) and (A7) of the paper, we find that

$$\frac{\partial p_2(p_1^{*o})}{\partial c} = \frac{(1 - \lambda) D'_n(p_1^{*o})}{2D'_2(p_2^{*o}) + p_2^{*o} D''_2(p_2^{*o}) + (1 - \lambda) D'_n(p_2^{*o} - c)} > 0,$$

where the inequality follows from $\phi'_2(\cdot) < 0$ by profit concavity ($\phi_2(\cdot)$ is defined by (A1) of the paper). As $d\bar{S}(p_1^{*o}, p_2^{*o}) / dp_1 < 0$ (see (A10) of the paper), the implicit function theorem implies that a sufficient (albeit not necessary) condition for $\partial p_1^{*o} / \partial c > 0$ is $\partial^2 \bar{S}(p_1^{*o}, p_2^{*o}) / \partial p_1 \partial c \leq 0$. Taking the derivative of $d\bar{S}(p_1, p_2) / dp_1$ in (A10) of the paper with respect to c yields after

some manipulation

$$\frac{\partial^2 \bar{S}(p_1^{*o}, p_2^{*o})}{\partial p_1 \partial c} = \frac{(1-\lambda)^2 D'_s(p_1^{*o})}{[\Pi_2''(p_2^{*o}) + (1-\lambda) D'_n(p_2^{*o} - c)]^2} \left\{ \Pi_2'''(p_2^{*o}) D'_n(p_2^{*o} - c) \right. \\ \left. \times \frac{\partial p_2(p_1^{*o})}{\partial c} - \Pi_2''(p_2^{*o}) D''_n(p_2^{*o} - c) \left[\frac{\partial p_2(p_1^{*o})}{\partial c} - 1 \right] \right\}.$$

As $\partial p_2(p_1^{*o})/\partial c \in (0, 1)$, sufficient (albeit not necessary) conditions for $\partial^2 \bar{S}(p_1^{*o}, p_2^{*o})/\partial p_1 \partial c \leq 0$ and thus $\partial p_1^{*o}/\partial c > 0$ are $\Pi_2'''(p_2^{*o}) \leq 0$ and $D''_n(p_2^{*o} - c) \leq 0$. Such conditions ensure that $d\bar{S}^{*o}/dc < 0$, which implies that there exists a unique threshold \bar{c}^o below which the dynamic storage solution applies in equilibrium. With linear demand functions $D_s(p_\tau) = \alpha_s - \beta p_\tau$ and $D_n(p_\tau) = \alpha_n - \beta p_\tau$, where $\alpha_s > \alpha_n$, we find that $d\bar{S}^{*o}/dc < 0$.

Now, we derive the condition ensuring that \bar{s}^{*o} increases with λ , which implies that \bar{c}^o increases with λ as well. Taking the derivative of $\bar{s}^{*o} = \bar{s}_n(p_1^{*o}(\lambda), \lambda)$ with respect to λ , we find that $d\bar{s}^{*o}/d\lambda = [d\bar{s}_n(p_1^{*o}, p_2^{*o})/dp_1] (\partial p_1^{*o}/\partial \lambda) + \partial \bar{s}^{*o}/\partial \lambda$. Applying the implicit function theorem to the expressions in (A3) and (A7) of the paper yields

$$\frac{\partial \bar{s}^{*o}}{\partial \lambda} = D'_n(p_2^{*o} - c) \frac{D_s(p_2^{*o}) - D_n(p_2^{*o}) + p_2^{*o} [D'_s(p_2^{*o}) - D'_n(p_2^{*o})]}{2D'_2(p_2^{*o}) + p_2^{*o} D''_2(p_2^{*o}) + (1-\lambda) D'_n(p_2^{*o} - c)} > 0,$$

where the inequality follows from $\phi'_2(\cdot) < 0$ by profit concavity ($\phi_2(\cdot)$ is defined by (A1) of the paper) and the assumptions about overestimating consumers. As $d\bar{s}_n(p_1^{*o}, p_2^{*o})/dp_1 < 0$ (see (A10) of the paper), we find that a sufficient (albeit not necessary) condition for $d\bar{s}^{*o}/d\lambda > 0$ is $\partial p_1^{*o}/\partial \lambda \leq 0$.

With linear demands $D_s(p_\tau) = \alpha_s - \beta p_\tau$ and $D_n(p_\tau) = \alpha_n - \beta p_\tau$, where $\alpha_s > \alpha_n$, we have $\bar{c}^o > 0$. Furthermore, we find that $d\bar{s}^{*o}/d\lambda > 0$ and $\partial \bar{c}^o/\partial \lambda > 0$. ■

Proof of Proposition 3. We start with underestimating consumers. With linear demands $D_s(p_\tau) = \alpha_s - \beta p_\tau$ and $D_n(p_\tau) = \alpha_n - \beta p_\tau$, where $\alpha_n > \alpha_s$, we find that $\partial p_\tau^{*u}/\partial \lambda > 0$, $\tau \in \{1, 2\}$, if and only if $c > (\alpha_n - 2\alpha_s) / (2\beta)$. By the proof of Proposition 1, this condition is equivalent to $p_1^{*u} < p_1^{sm}$. In the dynamic solution without storage (see the proof in point (IIb) of Lemma 1), we find from the expression in (A3) of the paper (with underestimating consumers) that $\partial p_\tau^{*u}/\partial \lambda < 0$, $\tau \in \{1, 2\}$. In the static solution, we know from the proof of Lemma 1 that p_1^{sm} is independent of λ and $\partial p_2^m/\partial \lambda < 0$ (see (A3) of the paper with underestimating consumers).

Now, we turn to the case of overestimating consumers. With linear demand functions $D_s(p_\tau) = \alpha_s - \beta p_\tau$ and $D_n(p_\tau) = \alpha_n - \beta p_\tau$, where $\alpha_s > \alpha_n$, we obtain that $\partial p_1^{*o}/\partial \lambda < 0$ when α_s is below a certain threshold. Otherwise, there exists a unique threshold $\lambda^o \in (0, 1)$ such that $\partial p_1^{*o}/\partial \lambda > 0$ if and only if $\lambda < \lambda^o$. Furthermore, it holds $\partial p_2^{*o}/\partial \lambda > 0$. In the dynamic solution without storage (see the proof of Lemma 4), we find from (A3) of the paper (with overestimating consumers) that $\partial p_1^{*o}/\partial \lambda = [1/D'_s(p_1^{*o})] D'_n(p_2^m - c) (\partial p_2^m/\partial \lambda) > 0$ (as $D'_s(\cdot) < 0$ and $D'_n(\cdot) < 0$) and $\partial p_2^{*o}/\partial \lambda > 0$. In the static solution, we know from the proof of Lemma 1 that p_1^{sm} is independent of λ and $\partial p_2^m/\partial \lambda > 0$ (see (A3) of the paper with overestimating consumers). ■

Proof of Lemma 5. This solution must also satisfy the condition that consumers with high

valuation do not store ex post, i.e., $D_1(p_1^*) + s(p_1^*) - D_h(p_1^*) \leq 0$, as implied by the condition for intertemporal marginal utilities in (2) of the paper (recall $p_2^* = p_1^* + c$). Clearly, this solution exists for some parameter constellations because the condition for intertemporal marginal utilities in (2) of the paper is more stringent for consumers with low valuation ($k = l$) than for consumers with high valuation ($k = h$). When condition (2) of the paper (for $k = h$) is violated at the candidate solution, we find that $\bar{s}_h(p_1, p_2^e)$ makes condition (2) of the paper (for $k = h$) binding and thus equalizes intertemporal marginal utilities for consumers with high valuation. Following the same logic as in the proof of Lemma 1, storage is harmful to the firm because it imposes a cost c for each unit bought at p_1 and stored in the first period instead of being bought at $p_2 = p_1 + c$. The firm can charge the first period equilibrium price such that $D_1(\tilde{p}_1^*) + s(\tilde{p}_1^*) - D_h(\tilde{p}_1^*) = 0$, which yields $\bar{s}_h(p_1, p_2^e) = 0$. When the second period demand $D_2(\cdot)$ rises substantially, the firm may find it unfeasible (or unprofitable) to prevent ex post storage even by consumers with high valuation. This implies that $\bar{s}_h(p_1, p_2^e) > 0$ is determined by the condition for the equalization of intertemporal marginal utilities (3) of the paper (for $k = h$). As shown by Dudine et al. (2006), the firm increases prices to dampen storage and thus the results in the proposition for $D_2(\cdot)$ high enough are reinforced.

With linear demands $D_h(p_\tau) = \alpha_h - \beta p_\tau$ and $D_l(p_\tau) = \alpha_l - \beta p_\tau$, which yield $D_1(p_1) = \theta\alpha_h + (1 - \theta)\alpha_l - \beta p_1$ and $D_2(p_2, \zeta) = \theta(\alpha_h + \zeta) + (1 - \theta)(\alpha_l + \zeta) - \beta p_2$, where $\alpha_h > \alpha_l$ and $\zeta \in \mathbb{R}$, the difference in the firm's profits between the solution with ex ante storage and the solution with only ex post storage increases with ζ . Hence, there exists a unique threshold $\hat{\zeta}$ such that the solution with ex ante storage is more profitable than the solution with only ex post storage if and only if $\zeta > \hat{\zeta}$, where $\hat{\zeta} > 0$. ■

Proof of Proposition 4. Using linear demand functions $D_h(p_\tau) = \alpha_h - \beta p_\tau$ and $D_l(p_\tau) = \alpha_l - \beta p_\tau$, which yield $D_1(p_1) = \theta\alpha_h + (1 - \theta)\alpha_l - \beta p_1$ and $D_2(p_2, \zeta) = \theta(\alpha_h + \zeta) + (1 - \theta)(\alpha_l + \zeta) - \beta p_2$, where $\alpha_h > \alpha_l$ and $\zeta \in \mathbb{R}$, we find that there exists a threshold $\bar{\zeta}$ such that it holds $p_1^* < p_1^m$ if and only if $\zeta < \bar{\zeta}$, with $\partial\bar{\zeta}/\partial\theta > 0$. ■

3 Longer time horizon

3.1 Finite horizon

We consider a finite horizon setting with $T \geq 2$ periods. The demand faced by the firm in period $\tau \in \{1, \dots, T\}$ corresponds to the demand for consumption $D_\tau(p_\tau)$ in period τ , reduced by the amount of ex post storage $\bar{S}_{\tau-1}(p_{\tau-1})$ inherited from period $\tau - 1$ and inflated by the amount of ex ante storage $S_\tau(p_\tau)$ in period τ , where $\bar{S}_1 = 0$ (as ex post storage is absent in the first period) and $S_T(p_T) = \bar{S}_T(p_T) = 0$ (as consumers do not store in the final period). In each period, a fraction $\lambda \in (0, 1)$ of the consumer population consists of sophisticated consumers that hold unbiased beliefs about their preferences and perfectly know their valuation for the good at the purchase stage. The remaining fraction $1 - \lambda$ is formed of naïve consumers that hold biased beliefs. In any period τ , consumers of type $i = s, n$ (where s stands for sophisticates and n for naïfs) may have either high utility $u_h(x_\tau)$ or low utility $u_l(x_\tau)$ from the consumption level x_τ . Utility $u_k(x_\tau)$ is increasing and concave in x_τ , i.e., $u'_k(x_\tau) > 0$ and $u''_k(x_\tau) < 0$, with $u_k(0) = 0$, for $k = h, l$. Furthermore, it holds $u'_h(\cdot) > u'_l(\cdot)$. Naïfs may either underestimate or

overestimate their preferences. In any period τ , naïfs underestimating their preferences have high utility $u_h(x_\tau) \triangleq u_n(x_\tau)$ from the consumption level x_τ but at the purchase stage believe that they share the same low utility $u_l(x_\tau) \triangleq u_s(x_\tau)$ as sophisticates. Naïfs overestimating their preferences have low utility $u_l(x_\tau) \triangleq u_n(x_\tau)$ from the consumption level x_τ but at the purchase stage believe that they share the same high utility $u_h(x_\tau) \triangleq u_s(x_\tau)$ as sophisticates. To focus on the most relevant results, we consider sufficiently small values of the storage cost such that storage arises in each period. The solution concept is Markov perfect equilibrium, where the firm's price p_τ in period τ is contingent only on the state of the game, namely, the amount of ex post storage $\bar{S}_{\tau-1}$ inherited from period $\tau - 1$.

We start with the case of naïve underestimating consumers. Following the logic as in the main analysis, only sophisticated consumers store ex post in period τ , i.e., $\bar{S}_\tau(p_\tau) = \lambda S_\tau(p_\tau)$. In the following remark, we characterize the equilibrium price dynamics in the presence of storage.

Remark 1 Consider a finite horizon setting with $T \geq 2$ periods. Also, suppose that naïve consumers underestimate their preferences. Then, in period $\tau \in \{1, \dots, T - 1\}$, the equilibrium price in the presence of storage exhibits the following features: (i) for $c < \tilde{c}_\tau^u$, it is higher than the static monopoly price if $\lambda \in (\bar{\lambda}_\tau^u, \tilde{\lambda}_\tau^u)$, and (ii) for $c \geq \tilde{c}_\tau^u$, it is lower than the static monopoly price. In period $\tau = T$, the equilibrium price in the presence of storage is lower than the static monopoly price.

Proof of Remark 1. Given the amount of ex post storage $\bar{S}_{\tau-1} = \lambda S_{\tau-1}$ inherited from period $\tau - 1$, the firm faces in period τ the following recursive maximization problem

$$Z_\tau(S_{\tau-1}) \triangleq \max_{p_\tau} p_\tau [D_\tau(p_\tau) - \lambda S_{\tau-1} + S_\tau(p_\tau)] + Z_{\tau+1}(S_\tau(p_\tau)). \quad (\text{S1})$$

The first-order condition for p_τ is given by

$$D_\tau(p_\tau) - \lambda S_{\tau-1} + S_\tau(p_\tau) + p_\tau [D'_\tau(p_\tau) + S'_\tau(p_\tau)] + \frac{\partial Z_{\tau+1}(S_\tau)}{\partial S_\tau} S'_\tau(p_\tau) = 0. \quad (\text{S2})$$

Applying the envelope theorem to the firm's maximization problem in (S1) rolled forward one period yields $\partial Z_{\tau+1}(S_\tau) / \partial S_\tau = -\lambda p_{\tau+1}(S_\tau)$. Furthermore, rolling forward one period the first-order condition (S2) and solving for $p_{\tau+1}(S_\tau)$, the implicit function theorem implies that $\partial p_{\tau+1}(S_\tau) / \partial S_\tau < 0$. Applying the implicit function theorem to the binding storability constraint $p_\tau + c = p_{\tau+1}(S_\tau)$ yields

$$S'_\tau(p_\tau) = \frac{1}{\partial p_{\tau+1}(S_\tau) / \partial S_\tau} < 0,$$

where the inequality follows from $\partial p_{\tau+1}(S_\tau) / \partial S_\tau < 0$. Then, the levels of ex ante storage $S_\tau(p_\tau)$ and ex post storage $\bar{S}_\tau(p_\tau) = \lambda S_\tau(p_\tau)$ decline with the price p_τ . To promote storage, the firm decreases the price. Using the binding storability constraint $p_\tau + c = p_{\tau+1}$ and recalling that $\partial Z_{\tau+1}(S_\tau) / \partial S_\tau = -\lambda p_{\tau+1}(S_\tau)$, the first-order condition for p_τ in (S2) can be rewritten as

$$D_\tau(p_\tau) + p_\tau D'_\tau(p_\tau) - \lambda S_{\tau-1} + S_\tau(p_\tau) + [p_\tau(1 - \lambda) - c\lambda] S'_\tau(p_\tau) = 0, \quad (\text{S3})$$

which identifies the equilibrium price p_τ^{*u} with storage in period τ for the case of underestimating consumers. Substituting the first-order condition for the static monopoly price p_τ^m in period τ , i.e., $D_\tau(p_\tau^m) + p_\tau^m D'_\tau(p_\tau^m) = 0$, into the left-hand side of the first-order condition for p_τ^{*u} in (S3) yields $-\lambda S_{\tau-1} + S_\tau(p_\tau^m) + [p_\tau^m(1-\lambda) - c\lambda] S'_\tau(p_\tau^m)$. First, suppose that the static solution is not feasible at least in one period, i.e., $c < \tilde{c}_\tau^u \triangleq p_{\tau+1}^m - p_\tau^m$ for some $\tau \in \{1, \dots, T-1\}$. As $S'_\tau(\cdot) < 0$, there exist two thresholds $\bar{\lambda}_\tau^u \triangleq p_\tau^m / (p_\tau^m + c)$ and $\tilde{\lambda}_\tau^u \triangleq S_\tau(p_\tau^m) / S_{\tau-1}$ such that a sufficient (albeit not necessary) condition for the aforementioned expression to be positive is $\lambda \in (\bar{\lambda}_\tau^u, \tilde{\lambda}_\tau^u)$, where $\lambda > \bar{\lambda}_\tau^u$ ensures that the expression in square brackets is negative and $\lambda < \tilde{\lambda}_\tau^u$ ensures that the sum of the first two terms is positive. Thus, for $c < \tilde{c}_\tau^u$, it holds $p_\tau^{*u} > p_\tau^m$ if $\lambda \in (\bar{\lambda}_\tau^u, \tilde{\lambda}_\tau^u)$, for $\tau \in \{1, \dots, T-1\}$. Now, suppose that the static solution is feasible in every period, i.e., $c \geq \tilde{c}_\tau^u$, for any $\tau \in \{1, \dots, T-1\}$. Sequential optimality implies that $p_T^{*u} < p_T^m$. Thus, it holds $p_{T-t}^{*u} = p_T^{*u} - tc < p_{T-t}^m$ for any $t \in \{1, \dots, T-1\}$, where the inequality follows from $c \geq \tilde{c}_\tau^u$ or equivalently $p_{T-t}^m + c \geq p_{T-t+1}^m$ for any $t \in \{1, \dots, T-1\}$. ■

Now, we turn to the case of naïve overestimating consumers. Following the same logic as in the main analysis, ex ante storage vanishes and only naïfs (with low valuation for the good) store ex post in period τ . The consumption level $D_\tau(p_\tau) - \bar{s}_{n\tau}(p_\tau, p_{\tau+1}^e)$ per naïve consumer equalizes marginal utilities across periods, i.e., $u'_n(D_\tau(p_\tau) - \bar{s}_{n\tau}(p_\tau, p_{\tau+1}^e)) + c = u'_n(D_n(p_{\tau+1}^e))$. As $u'_n(D_n(p_{\tau+1}^e)) = p_{\tau+1}^e$, the condition for the equalization of intertemporal marginal utilities (equation (3) of the paper) becomes

$$\bar{s}_{n\tau}(p_\tau, p_{\tau+1}^e) = D_\tau(p_\tau) - D_n(p_{\tau+1}^e - c). \quad (\text{S4})$$

We obtain the following results.

Remark 2 Consider a finite horizon setting with $T \geq 2$ periods. Also, suppose that naïve consumers overestimate their preferences. Then, in period $\tau \in \{1, \dots, T-1\}$, for λ high enough, the equilibrium price in the presence of storage is higher than the static monopoly price. In period $\tau = T$, the equilibrium price in the presence of storage is lower than the static monopoly price.

Proof of Remark 2. Given the level of ex post storage $\bar{S}_{\tau-1} = (1-\lambda)\bar{s}_{n(\tau-1)}$ inherited from period $\tau-1$, the firm faces in period τ the following recursive maximization problem

$$Z_\tau(\bar{S}_{\tau-1}) \triangleq \max_{p_\tau} p_\tau [D_\tau(p_\tau) - \bar{S}_{\tau-1}] + Z_{\tau+1}(\bar{S}_\tau(p_\tau, p_{\tau+1}^e)). \quad (\text{S5})$$

As $p_{\tau+1}^e = p_{\tau+1}$, the first-order condition for p_τ is given by

$$D_\tau(p_\tau) - \bar{S}_{\tau-1} + p_\tau D'_\tau(p_\tau) + \frac{\partial Z_{\tau+1}(\bar{S}_\tau)}{\partial \bar{S}_\tau} \frac{d\bar{S}_\tau(p_\tau, p_{\tau+1})}{dp_\tau} = 0. \quad (\text{S6})$$

Applying the envelope theorem to the firm's maximization problem in (S5) rolled forward one period yields $\partial Z_{\tau+1}(\bar{S}_\tau) / \partial \bar{S}_\tau = -p_{\tau+1}(\bar{S}_\tau)$. Furthermore, applying the implicit function theorem to (S4) and (S6) rolled forward one period, we find that

$$\frac{d\bar{S}_\tau(p_\tau, p_{\tau+1})}{dp_\tau} = \frac{(1-\lambda) D'_\tau(p_\tau) \Phi(p_{\tau+1})}{\Phi(p_{\tau+1}) + (1-\lambda) D'_n(p_{\tau+1} - c)} < 0,$$

where $\Phi(\cdot) \triangleq 2D'_{\tau+1}(\cdot) + p_{\tau+1}D''_{\tau+1}(\cdot) + \partial^2 Z_{\tau+2}(\cdot) / \partial p_{\tau+1}^2 < 0$ (by profit concavity). Recalling that $\partial Z_{\tau+1}(\bar{S}_\tau) / \partial \bar{S}_\tau = -p_{\tau+1}(\bar{S}_\tau)$, the first-order condition for p_τ in (S6) can be rewritten as

$$D_\tau(p_\tau) + p_\tau D'_\tau(p_\tau) - \bar{S}_{\tau-1} - p_{\tau+1} \frac{d\bar{S}_\tau(p_\tau, p_{\tau+1})}{dp_\tau} = 0, \quad (\text{S7})$$

which identifies the equilibrium price p_τ^{*o} with storage in period τ for the case of overestimating consumers. Substituting the first-order condition for the static monopoly price p_τ^m in period τ , i.e., $D_\tau(p_\tau^m) + p_\tau^m D'_\tau(p_\tau^m) = 0$, into the left-hand side of the first-order condition for p_τ^{*o} in (S7) yields $-\bar{S}_{\tau-1} - p_{\tau+1} d\bar{S}_\tau(p_\tau^m, p_{\tau+1}) / dp_\tau$. As $d\bar{S}_\tau(\cdot) / dp_\tau < 0$ and $\bar{S}_{\tau-1} = (1 - \lambda) \bar{s}_{n(\tau-1)}$, this expression is positive for λ high enough, which ensures that $\bar{S}_{\tau-1}$ is not too large. In this case, it holds $p_\tau^{*o} > p_\tau^m$ for $\tau \in \{1, \dots, T-1\}$. Furthermore, sequential optimality implies that $p_T^{*o} < p_T^m$. ■

3.2 Infinite horizon

We now consider an infinite horizon setting, where consumer preferences are stochastic and follow a (stationary) Markov process. As in a finite horizon setting, the solution concept is Markov perfect equilibrium, where the firm's price p_τ in period τ is contingent only on the state of the game, namely, the amount of ex post storage $\bar{S}_{\tau-1}$ inherited from period $\tau-1$. At the purchase stage in period τ , with probability $\theta \in (0, 1)$ consumers expect high utility $u_h(x_\tau)$ and with complementary probability $1 - \theta$ they expect low utility $u_l(x_\tau)$ from the consumption level x_τ . Utility $u_k(x_\tau)$ is increasing and concave in x_τ , i.e., $u'_k(x_\tau) > 0$ and $u''_k(x_\tau) < 0$, with $u_k(0) = 0$, for $k = h, l$. Furthermore, it holds $u'_h(\cdot) > u'_l(\cdot)$. For any given price p_τ in period τ and expected price $p_{\tau+1}^e$ in period $\tau+1$, we denote by $s_\tau(p_\tau, p_{\tau+1}^e)$ the amount of ex ante storage for each consumer (which is identical across consumers) and by $\bar{s}_{k\tau}(p_\tau, p_{\tau+1}^e)$ the amount of ex post storage per consumer of type $k = h, l$ in period τ . In line with the main analysis, we have $\bar{s}_{h\tau}(p_\tau, p_{\tau+1}^e) = 0$ because consumers with high valuation do not store ex post. Furthermore, after learning their preferences, consumers with low valuation choose the amount of consumption that equalizes their marginal utilities across periods. Given the quantity $D_\tau(p_\tau, \zeta_\tau) + s_\tau(p_\tau, p_{\tau+1}^e)$ purchased in period τ , the level of consumption $D_\tau(p_\tau, \zeta_\tau) + s_\tau(p_\tau, p_{\tau+1}^e) - \bar{s}_{l\tau}(p_\tau, p_{\tau+1}^e)$ per consumer with low valuation in period τ is such that $u'_l(D_\tau(p_\tau, \zeta_\tau) + s_\tau(p_\tau, p_{\tau+1}^e) - \bar{s}_{l\tau}(p_\tau, p_{\tau+1}^e)) + c = \delta u'_l(D_l(p_{\tau+1}^e))$, where the parameter ζ_τ captures the magnitude of demand in period τ , i.e., $\partial D_\tau / \partial \zeta_\tau > 0$, and $\delta \in (0, 1)$ represents the discount factor. As $u'_l(D_l(p_{\tau+1}^e)) = p_{\tau+1}^e$,² the condition for the equalization of intertemporal marginal utilities (equation (3) of the paper) becomes

$$\bar{s}_{l\tau}(p_\tau, p_{\tau+1}^e) = D_\tau(p_\tau, \zeta_\tau) + s_\tau(p_\tau, p_{\tau+1}^e) - D_l(\delta p_{\tau+1}^e - c). \quad (\text{S8})$$

To focus on the most relevant results, we consider sufficiently small values of the storage cost such that storage arises in each period. In the following remark, we characterize the equilibrium price dynamics.

²Hence, nothing changes if consumer preferences vary across periods.

Remark 3 Consider an infinite horizon setting. In the presence of ex ante storage, the equilibrium price in period τ is lower (higher) than the static monopoly price if $\zeta_{\tau+1}$ is low (high) enough and θ is high (low) enough. In the presence of only ex post storage, the equilibrium price in period τ is higher than the static monopoly price if θ is high enough.

Proof of Remark 3. Given the amount of ex post storage $\bar{S}_{\tau-1} = (1 - \theta) \bar{s}_{l(\tau-1)}$ inherited from period $\tau - 1$ by consumers with low valuation, the firm faces in period τ the following recursive maximization problem

$$Z(\bar{S}_{\tau-1}) \triangleq \max_{p_\tau} p_\tau [D_\tau(p_\tau, \zeta_\tau) - \bar{S}_{\tau-1} + S_\tau(p_\tau, p_{\tau+1}^e)] + \delta \mathbb{E} [Z(\bar{S}_\tau(p_\tau, p_{\tau+1}^e))]. \quad (\text{S9})$$

First, suppose that the storability constraint is binding in period τ , i.e., $p_\tau + c = \delta p_{\tau+1}$. We find from (S8), where $S_\tau(p_\tau, p_{\tau+1}^e) = s_\tau(p_\tau, p_{\tau+1}^e)$ and $\bar{S}_\tau(p_\tau, p_{\tau+1}^e) = (1 - \theta) \bar{s}_{l\tau}(p_\tau, p_{\tau+1}^e)$, that the firm's maximization problem becomes

$$Z(\bar{S}_{\tau-1}) \triangleq \max_{p_\tau} p_\tau \left[D_l(p_\tau) - \bar{S}_{\tau-1} + \frac{\bar{S}_\tau(p_\tau)}{1 - \theta} \right] + \delta \mathbb{E} [Z(\bar{S}_\tau(p_\tau))]. \quad (\text{S10})$$

The first-order condition for p_τ is given by

$$D_l(p_\tau) - \bar{S}_{\tau-1} + \frac{\bar{S}_\tau(p_\tau)}{1 - \theta} + p_\tau \left[D_l'(p_\tau) + \frac{\bar{S}'_\tau(p_\tau)}{1 - \theta} \right] + \delta \mathbb{E} \left[\frac{\partial Z(\bar{S}_\tau)}{\partial \bar{S}_\tau} \bar{S}'_\tau(p_\tau) \right] = 0. \quad (\text{S11})$$

Applying the envelope theorem to the firm's maximization problem in (S10) rolled forward one period yields $\partial Z(\bar{S}_\tau) / \partial \bar{S}_\tau = -p_{\tau+1}(\bar{S}_\tau)$. Furthermore, rolling forward one period the first-order condition (S11) and solving for $p_{\tau+1}(\bar{S}_\tau)$, we find from the implicit function theorem that $\partial p_{\tau+1}(\bar{S}_\tau) / \partial \bar{S}_\tau < 0$. Applying the implicit function theorem to the binding storability constraint $p_\tau + c = \delta p_{\tau+1}(\bar{S}_\tau)$ yields

$$\bar{S}'_\tau(p_\tau) = \frac{1/\delta}{\partial p_{\tau+1}(\bar{S}_\tau) / \partial \bar{S}_\tau} < 0,$$

where the inequality follows from $\partial p_{\tau+1}(\bar{S}_\tau) / \partial \bar{S}_\tau < 0$. Using the binding storability constraint $p_\tau + c = \delta p_{\tau+1}$ and recalling that $\partial Z(\bar{S}_\tau) / \partial \bar{S}_\tau = -p_{\tau+1}(\bar{S}_\tau)$, the first-order condition for p_τ in (S11) can be rewritten as

$$D_l(p_\tau) + p_\tau D_l'(p_\tau) - \bar{S}_{\tau-1} + \frac{\bar{S}_\tau(p_\tau)}{1 - \theta} + [\theta p_\tau - (1 - \theta)c] \frac{\bar{S}'_\tau(p_\tau)}{1 - \theta} = 0, \quad (\text{S12})$$

which identifies the equilibrium price p_τ^* with ex ante storage in period τ . Evaluating the left-hand side of the first-order condition for p_τ^* in (S12) at p_τ^m and multiplying by $1 - \theta$, we obtain the following expression

$$(1 - \theta) [D_l(p_\tau^m) + p_\tau^m D_l'(p_\tau^m) - \bar{S}_{\tau-1}] + \bar{S}_\tau(p_\tau^m) + [\theta p_\tau^m - (1 - \theta)c] \bar{S}'_\tau(p_\tau^m),$$

where $\bar{S}'_\tau(\cdot) < 0$. The expression in the second square brackets is positive if θ is high enough.

For $\zeta_{\tau+1}$ low enough, we have $p_{\tau+1}^m < p_\tau^m + c$, which implies $\bar{S}_\tau(p_\tau^m) < 0$. As for θ high enough the first term is relatively small, we find that, for $\zeta_{\tau+1}$ low enough and θ high enough, the entire expression is negative and thus $p_\tau^* < p_\tau^m$. Furthermore, for θ low enough, the expression in the second square brackets is negative. As there are sufficiently large values of $\zeta_{\tau+1}$ such that $\bar{S}_\tau(p_\tau^m) > 0$ and the sum of the first two terms is positive, we find that, for $\zeta_{\tau+1}$ high enough and θ low enough, the entire expression is positive and thus $p_\tau^* > p_\tau^m$.

Now, we turn to the case where the storability constraint is not binding in period τ , i.e., $p_\tau + c > \delta p_{\tau+1}$. As $S_\tau(p_\tau, p_{\tau+1}^e) = 0$, the firm's maximization problem becomes

$$Z(\bar{S}_{\tau-1}) \triangleq \max_{p_\tau} p_\tau [D_\tau(p_\tau, \zeta_\tau) - \bar{S}_{\tau-1}] + \delta \mathbb{E} [Z(\bar{S}_\tau(p_\tau, p_{\tau+1}))]. \quad (\text{S13})$$

The first-order condition for p_τ is given by

$$D_\tau(p_\tau, \zeta_\tau) - \bar{S}_{\tau-1} + p_\tau D'_\tau(p_\tau, \zeta_\tau) + \delta \mathbb{E} \left[\frac{\partial Z(\bar{S}_\tau)}{\partial \bar{S}_\tau} \frac{d\bar{S}_\tau(p_\tau, p_{\tau+1})}{dp_\tau} \right] = 0. \quad (\text{S14})$$

Applying the envelope theorem to the firm's maximization problem in (S13) rolled forward one period yields $\partial Z(\bar{S}_\tau) / \partial \bar{S}_\tau = -p_{\tau+1}(\bar{S}_\tau)$. Furthermore, applying the implicit function theorem to (S8) and (S14) rolled forward one period, we find that

$$\frac{d\bar{S}_\tau(p_\tau, p_{\tau+1})}{dp_\tau} = \frac{(1-\theta) D'_\tau(p_\tau, \zeta_\tau) \Phi(p_{\tau+1})}{\Phi(p_{\tau+1}) + \delta(1-\theta) D'_n(\delta p_{\tau+1} - c)} < 0,$$

where $\Phi(\cdot) \triangleq 2D'_{\tau+1}(\cdot) + p_{\tau+1} D''_{\tau+1}(\cdot) + \delta \mathbb{E} [\partial^2 Z(\cdot) / \partial p_{\tau+1}^2] < 0$ (by profit concavity). Recalling that $\partial Z(\bar{S}_\tau) / \partial \bar{S}_\tau = -p_{\tau+1}(\bar{S}_\tau)$, the first-order condition for p_τ in (S14) can be rewritten as

$$D_\tau(p_\tau, \zeta_\tau) + p_\tau D'_\tau(p_\tau, \zeta_\tau) - \bar{S}_{\tau-1} - \delta p_{\tau+1} \frac{d\bar{S}_\tau(p_\tau, p_{\tau+1})}{dp_\tau} = 0, \quad (\text{S15})$$

which identifies the equilibrium price p_τ^{**} with only ex post storage in period τ . Substituting the first-order condition for the static monopoly price p_τ^m in period τ , i.e., $D_\tau(p_\tau^m, \zeta_\tau) + p_\tau^m D'_\tau(p_\tau^m, \zeta_\tau) = 0$, into the left-hand side of the first-order condition for p_τ in (S15) yields $-\bar{S}_{\tau-1} - \delta p_{\tau+1} d\bar{S}_\tau(p_\tau^m, p_{\tau+1}) / dp_\tau$. As $d\bar{S}_\tau(\cdot) / dp_\tau < 0$ and $\bar{S}_{\tau-1} = (1-\theta) \bar{s}_{l(\tau-1)}$, this expression is positive for θ high enough, which ensures that $\bar{S}_{\tau-1}$ is not too large. In this case, it holds $p_\tau^{**} > p_\tau^m$. ■

4 Nonlinear pricing

We examine the case where the firm offers consumers a nonlinear pricing policy and identify the main features of the dynamic storage solution. We also compare the dynamic storage solution with the static solution in terms of consumer surplus. Our main results are formalized in the following remark.

Remark 4 *A. With naïve underestimating consumers, in the dynamic storage solution, sophisticated consumers obtain nonnegative surplus and naïve consumers obtain positive surplus. If the share of*

naïve consumers is large enough, sophisticated consumers are as well off and naïve consumers are better off with respect to the static solution for sufficiently small values of the storage cost.

B. With naïve overestimating consumers, in the dynamic storage solution, sophisticated consumers obtain nonnegative surplus and naïve consumers may obtain negative surplus. If the share of sophisticated consumers is large enough, sophisticated consumers are at most as well off and naïve consumers are worse off with respect to the static solution for sufficiently small values of the storage cost.

Proof of Remark 4. The firm offers a pricing policy $\{Q_{i\tau}, P_{i\tau}\}$ that specifies a quantity $Q_{i\tau}$ and a payment $P_{i\tau}$ for consumers of type $i = s, n$ (where s stands for sophisticates and n for naïfs) in period $\tau \in \{1, 2\}$. As in the baseline model, a share $\lambda \in (0, 1)$ of the consumer population consists of sophisticated consumers and the remaining share $1 - \lambda$ is formed of naïve consumers. Type i 's consumers have utility $u_i(x_\tau)$ from the consumption level x_τ , which is increasing and concave in x_τ , i.e., $u_i'(x_\tau) > 0$ and $u_i''(x_\tau) < 0$, with $u_i(0) = 0$. To ensure the concavity of the firm's profit function, we introduce a (constant) unit cost of production $\gamma > 0$.³

We start with the case of naïve underestimating consumers, where $u_n'(\cdot) > u_s'(\cdot)$. By the revelation principle, the firm designs a contract menu $\{(Q_{s2}, P_{s2}), (Q_{n2}, P_{n2})\}$ in the second period, where (Q_{s2}, P_{s2}) denotes the contract designed for sophisticates and (Q_{n2}, P_{n2}) denotes the contract designed for naïfs. Proceeding backward, the firm's second period maximization problem writes as

$$\max_{\{(Q_{s2}, P_{s2}), (Q_{n2}, P_{n2})\}} \lambda (P_{s2} - \gamma Q_{s2}) + (1 - \lambda) (P_{n2} - \gamma Q_{n2}) \quad (\text{S16})$$

subject to the following participation and incentive constraints

$$u_s(Q_{s2} + \bar{S}_s) - P_{s2} \geq u_s(\bar{S}_s) \quad (\text{S17})$$

$$u_n(Q_{n2} + \bar{S}_n) - P_{n2} \geq u_n(\bar{S}_n) \quad (\text{S18})$$

$$u_s(Q_{s2} + \bar{S}_s) - P_{s2} \geq u_s(Q_{n2} + \bar{S}_s) - P_{n2} \quad (\text{S19})$$

$$u_n(Q_{n2} + \bar{S}_n) - P_{n2} \geq u_n(Q_{s2} + \bar{S}_n) - P_{s2}, \quad (\text{S20})$$

where \bar{S}_i represents the amount of ex post storage inherited from the first period by consumers of type $i = s, n$. In line with our main analysis, as all consumers believe that they have the same preferences at first period purchase stage but naïfs discover high valuation for the good, naïfs consume (weakly) more than sophisticates in the first period. This implies that the level of ex post storage is (weakly) lower for naïfs than for sophisticates, i.e., $\bar{S}_n \leq \bar{S}_s$. The participation constraints (S17) and (S18) ensure that consumers of type $i = s, n$ are willing to accept the second period contract when obtaining at least the utility $u_i(\cdot)$ associated with consumption of the quantity stored \bar{S}_i . The incentive constraints (S19) and (S20) induce consumers to select the contract intended for their type instead of the one intended for the other type. Following a standard approach, we show that, as naïfs have higher valuation for the good, the participation

³Our findings are qualitatively unaffected in the absence of any cost of production, with the main difference that the quantity offered can be at the highest possible level (for instance, due to capacity constraints).

constraint (S17) for sophisticates and the incentive constraint (S20) for naïfs are binding in equilibrium, whereas the participation constraint (S18) for naïfs and the incentive constraint (S19) for sophisticates are slack. Combining (S17) and (S20) yields $u_n(Q_{n2} + \bar{S}_n) - P_{n2} - u_n(\bar{S}_n) \geq u_n(Q_{s2} + \bar{S}_n) - P_{s2} - u_n(\bar{S}_n) \geq u_s(Q_{s2} + \bar{S}_s) - P_{s2} - u_s(\bar{S}_s) \geq 0$, where the second inequality follows from $u'_n(\cdot) > u'_s(\cdot)$ and $u''_i(\cdot) < 0$, for $i = s, n$ (recall $\bar{S}_n \leq \bar{S}_s$). Hence, constraint (S18) is implied by constraints (S17) and (S20). Furthermore, constraints (S17) and (S20) must be binding in equilibrium, otherwise the firm could increase consumers' payments and be better off.⁴ Substituting the binding constraints (S17) and (S20) into the firm's second period maximization problem in (S16) and taking the first-order conditions for $Q_{s2} \geq 0$ and $Q_{n2} \geq 0$ yields

$$u'_s(Q_{s2} + \bar{S}_s) - (1 - \lambda) u'_n(Q_{s2} + \bar{S}_n) - \lambda\gamma \leq 0 \quad (\text{S21})$$

and

$$u'_n(Q_{n2} + \bar{S}_n) - \gamma \leq 0, \quad (\text{S22})$$

where the equalities hold for $Q_{s2} > 0$ and $Q_{n2} > 0$, respectively. In equilibrium, it holds $Q_{n2} \geq Q_{s2}$. As $u'_n(\cdot) > u'_s(\cdot)$, the quantity Q_{s2} for sophisticates is such that their consumption $Q_{s2} + \bar{S}_s$ is distorted below the efficient level, i.e., $u'_s(Q_{s2} + \bar{S}_s) - \gamma > 0$. The quantity Q_{n2} for naïfs ensures that for $Q_{n2} > 0$ their consumption $Q_{n2} + \bar{S}_n$ is efficient, i.e., $u'_n(Q_{n2} + \bar{S}_n) - \gamma = 0$. Notably, sophisticates' consumption can be efficient even for $Q_{n2} = 0$ because they resort to the quantity stored.⁵ Hence, given the first period pricing policy and consumers' storage decisions, the second period pricing policy is $\{(Q_{s2}, P_{s2}), (Q_{n2}, P_{n2})\}$, where Q_{s2} and Q_{n2} respectively satisfy (S21) and (S22), whereas P_{s2} and P_{n2} follow from the binding constraints (S17) and (S20). It holds $Q_{s2} = P_{s2} = 0$ for $\lambda \leq [u'_n(\bar{S}_n) - u'_s(\bar{S}_s)] / [u'_n(\bar{S}_n) - \gamma]$.

Given the first period pricing policy (Q_{s1}, P_{s1}) and the second period (expected) pricing policy (Q_{s2}, P_{s2}) , sophisticates' ex ante and ex post storage $S_s = \bar{S}_s \geq 0$ writes as

$$S_s = \bar{S}_s = \arg \max_{\tilde{S}_s} u_s(Q_{s1} - \tilde{S}_s) + u_s(Q_{s2} + \tilde{S}_s) - P_{s1} - P_{s2} - c\tilde{S}_s. \quad (\text{S23})$$

This yields $u'_s(Q_{s2} + S_s) - u'_s(Q_{s1} - S_s) - c \leq 0$, where the equality holds for $S_s > 0$.⁶ Naïfs' ex ante storage is also equal to S_s because naïfs believe that they have the same preferences as sophisticates at the first period purchase stage. Given the first period pricing policy (Q_{s1}, P_{s1}) and the second period (expected) pricing policy (Q_{n2}, P_{n2}) , naïfs' ex post storage $\bar{S}_n \geq 0$ writes as

$$\bar{S}_n = \arg \max_{\tilde{S}_n} u_n(Q_{s1} - \tilde{S}_n) + u_n(Q_{n2} + \tilde{S}_n) - P_{n2} - c\tilde{S}_n. \quad (\text{S24})$$

⁴Constraint (S19) is to be checked ex post.

⁵Substituting the binding constraints (S17) and (S20) into constraint (S19) yields after some manipulation $u_n(Q_{n2} + \bar{S}_n) - u_n(Q_{s2} + \bar{S}_n) \geq u_s(Q_{n2} + \bar{S}_s) - u_s(Q_{s2} + \bar{S}_s)$, where the inequality follows from $Q_{n2} \geq Q_{s2}$ as well as $u'_n(\cdot) > u'_s(\cdot)$ and $u''_i(\cdot) < 0$, for $i = s, n$ (recall $\bar{S}_n \leq \bar{S}_s$).

⁶As consumers' expectations are correct in equilibrium, with a small abuse of notation we omit the expectation operator in the second period pricing policy.

This yields $u'_n(Q_{n2} + \bar{S}_n) - u'_n(Q_{s1} - \bar{S}_n) - c \leq 0$, where the equality holds for $\bar{S}_n > 0$.

In anticipation of the second period pricing policy $\{(Q_{s2}, P_{s2}), (Q_{n2}, P_{n2})\}$ (as implied by (S16) with $u'_n(\cdot) > u'_s(\cdot)$) and consumers' storage decisions (as implied by (S23) and (S24)), the firm's first period maximization problem is given by

$$\max_{\{Q_{s1}, P_{s1}\}} P_{s1} - \gamma Q_{s1} + \lambda (P_{s2} - \gamma Q_{s2}) + (1 - \lambda) (P_{n2} - \gamma Q_{n2}) \quad (\text{S25})$$

subject to the following participation and non-skipping constraints

$$u_s(Q_{s1} - S_s) - cS_s - P_{s1} + u_s(Q_{s2} + S_s) - P_{s2} \geq 0 \quad (\text{S26})$$

$$u_s(Q_{s1} - S_s) - cS_s - P_{s1} + u_s(Q_{s2} + S_s) - P_{s2} \geq u_s(Q_{s2}) - P_{s2}. \quad (\text{S27})$$

The participation constraint (S26) ensures that all consumers, believing that they share the same utility $u_s(\cdot)$, are willing to accept the first period contract $\{Q_{s1}, P_{s1}\}$, in anticipation of the second period contract $\{Q_{s2}, P_{s2}\}$. The non-skipping constraint (S27) ensures that consumers do not have any incentives to remain idle in the first period by skipping their purchases and buying in the second period. We show that the non-skipping constraint (S27) is binding in equilibrium.⁷ Proceeding by contradiction, suppose that the non-skipping constraint (S27) is slack and thus $u_s(Q_{s2}) - P_{s2} < 0$. As the participation constraint (S17) yields $P_{s2} \leq u_s(Q_{s2} + S_s) - u_s(S_s)$, it must hold $u_s(Q_{s2}) < u_s(Q_{s2} + S_s) - u_s(S_s)$. However, this violates $u_s(Q_{s2}) + u_s(S_s) \geq u_s(Q_{s2} + S_s)$ (by subadditivity of $u_i(\cdot)$, for $i = s, n$). Hence, the non-skipping constraint (S27) is binding in equilibrium. Taking the first-order condition for Q_{s1} associated with the firm's maximization problem in (S25) yields after some manipulation

$$\begin{aligned} & u'_s(Q_{s1} - S_s) + Q'_{s2}(Q_{s1}) [u'_s(Q_{s2} + S_s) - u'_s(Q_{s2})] + S'_s(Q_{s1}) [u'_s(Q_{s2} + S_s) - u'_s(Q_{s1} - S_s) \\ & - c] + \lambda \{ u'_s(Q_{s2} + S_s) [Q'_{s2}(Q_{s1}) + S'_s(Q_{s1})] - u'_s(S_s) S'_s(Q_{s1}) - \gamma Q'_{s2}(Q_{s1}) \} \\ & + (1 - \lambda) \{ u'_n(Q_{n2} + \bar{S}_n) [Q'_{n2}(Q_{s1}) + \bar{S}'_n(Q_{s1})] - u'_n(Q_{s2} + \bar{S}_n) [Q'_{s2}(Q_{s1}) + \bar{S}'_n(Q_{s1})] \\ & + u'_s(Q_{s2} + S_s) [Q'_{s2}(Q_{s1}) + S'_s(Q_{s1})] - u'_s(S_s) S'_s(Q_{s1}) - \gamma Q'_{n2}(Q_{s1}) \} \\ & - \mu \{ u''_s(Q_{s2} + S_s) [Q'_{s2}(Q_{s1}) + S'_s(Q_{s1})] - u''_s(Q_{s1} - S_s) [1 - S'_s(Q_{s1})] \} \\ & - \nu \{ u''_n(Q_{n2} + \bar{S}_n) [Q'_{n2}(Q_{s1}) + \bar{S}'_n(Q_{s1})] - u''_n(Q_{s1} - \bar{S}_n) [1 - \bar{S}'_n(Q_{s1})] \} \\ & - \varphi \{ u''_s(Q_{s2} + S_s) [Q'_{s2}(Q_{s1}) + S'_s(Q_{s1})] - (1 - \lambda) u''_n(Q_{s2} + \bar{S}_n) [Q'_{s2}(Q_{s1}) + \bar{S}'_n(Q_{s1})] \} \\ & - \psi u''_n(Q_{n2} + \bar{S}_n) [Q'_{n2}(Q_{s1}) + \bar{S}'_n(Q_{s1})] - \gamma = 0, \end{aligned} \quad (\text{S28})$$

where $\mu \geq 0$ and $\nu \geq 0$ represent the Kuhn-Tucker multipliers associated with consumers' storage decisions (see (S23) and (S24)), and $\varphi \geq 0$ and $\psi \geq 0$ denote the Kuhn-Tucker multipliers associated with the first-order conditions (S21) and (S22). Thus, in the presence of naïve underestimating consumers, the pricing policy in the dynamic storage solution is given by (i) the first period contract $(Q_{s1}^{*u}, P_{s1}^{*u})$, where Q_{s1}^{*u} and P_{s1}^{*u} respectively satisfy the first-order condition (S28) and the binding non-skipping constraint (S27), and (ii) the second period contract

⁷Clearly, one constraint must be binding in equilibrium, otherwise the firm could increase the first period payment and be better off.

menu $\{(Q_{s2}^{*u}, P_{s2}^{*u}), (Q_{n2}^{*u}, P_{n2}^{*u})\}$, which stems from the firm's maximization problem in (S16) (where $u'_n(\cdot) > u'_s(\cdot)$), with the associated first-order conditions (S21) and (S22). The equilibrium levels of storage are S_s^{*u} for sophisticated consumers and \bar{S}_n^{*u} for naïve consumers (as implied by (S23) and (S24)). The dynamic storage solution with naïve underestimating consumers requires that (at least) sophisticates engage in storage activities, i.e., $S_s^{*u} > 0$, which emerges for a sufficiently low storage cost, i.e., $c < u'_s(Q_{s2}|_{S_s=0}) - u'_s(Q_{s1}|_{S_s=0})$ (see (S23)).

Using the binding constraint (S27), sophisticated consumers obtain the following surplus

$$\Psi_s^{*u} = u_s(Q_{s2}^{*u}) - P_{s2}^{*u} = u_s(Q_{s2}^{*u}) + u_s(S_s^{*u}) - u_s(Q_{s2}^{*u} + S_s^{*u}) \geq 0,$$

where the equality holds if and only if $Q_{s2}^{*u} = P_{s2}^{*u} = 0$. Naïve consumers obtain

$$\begin{aligned} \Psi_n^{*u} &= u_n(Q_{s1}^{*u} - \bar{S}_n^{*u}) - c\bar{S}_n^{*u} - P_{s1}^{*u} + u_n(Q_{n2}^{*u} + \bar{S}_n^{*u}) - P_{n2}^{*u} \\ &= u_n(Q_{s1}^{*u} - \bar{S}_n^{*u}) + u_n(Q_{s2}^{*u} + \bar{S}_n^{*u}) - u_s(Q_{s1}^{*u} - S_s^{*u}) \\ &\quad - 2u_s(Q_{s2}^{*u} + S_s^{*u}) + u_s(S_s^{*u}) + u_s(Q_{s2}^{*u}) + c(S_s^{*u} - \bar{S}_n^{*u}) > 0, \end{aligned}$$

where the inequality holds because naïve consumers (with higher valuation for the good) must obtain higher utility than sophisticates.

In the static solution, the pricing policy is characterized by (i) the first period contract (Q_{s1}^m, P_{s1}^m) , where Q_{s1}^m is such that $u'_s(Q_{s1}^m) - \gamma = 0$ and $P_{s1}^m = u_s(Q_{s1}^m)$, and (ii) the second period contract menu $\{(Q_{s2}^m, P_{s2}^m), (Q_{n2}^m, P_{n2}^m)\}$, which arises from the firm's maximization problem in (S16) (where $u'_n(\cdot) > u'_s(\cdot)$), with the associated first-order conditions (S21) and (S22) for $\bar{S}_s = \bar{S}_n = 0$. Note that the static solution can only emerge in the absence of storage, which occurs for sufficiently large values of the storage cost, i.e., $c \geq u'_s(Q_{s2}|_{S_s=0}) - u'_s(Q_{s1}|_{S_s=0})$ (see above). Sophisticated consumers obtain $\Psi_s^m = 0$ and naïve consumers obtain $\Psi_n^m = u_n(Q_{s1}^m) + u_n(Q_{s2}^m) - u_s(Q_{s1}^m) - u_s(Q_{s2}^m) > 0$.

We now compare the dynamic storage solution and the static solution in terms of consumer surplus. For the sake of convenience, we focus on the case where the share of naïfs is large enough, i.e., $\lambda \leq \tilde{\lambda}_{nl}^u \triangleq [u'_n(\bar{S}_n^{*u}) - u'_s(S_s^{*u})] / [u'_n(\bar{S}_n^{*u}) - \gamma]$ (see the discussion after (S22)). This implies that $Q_{s2}^{*u} = P_{s2}^{*u} = 0$. Then, it holds $\Psi_s^{*u} = \Psi_s^m = 0$. Furthermore, we show that $\Psi_n^{*u} > \Psi_n^m$, provided that the storage cost c is low enough. To proceed in a insightful manner, we fix $c = 0$. Our results apply for c low enough. First, we prove that the firm cannot benefit from $Q_{n2} > 0$. It follows from the first-order condition for Q_{s1} in (S28) that the impact of Q_{s1} on the firm's profit Π through Q_{n2} evaluated at $Q_{n2} = 0$ is given by

$$\begin{aligned} \left. \frac{\partial \Pi}{\partial Q_{s1}} \right|_{Q_{n2}=0} &= (1 - \lambda) \left\{ u'_n(\bar{S}_n) \left[Q'_{n2}(Q_{s1}) + \bar{S}'_n(Q_{s1}) \right] - u'_n(\bar{S}_n) \bar{S}'_n(Q_{s1}) - \gamma Q'_{n2}(Q_{s1}) \right\} \\ &\quad - \nu \left\{ u''_n(\bar{S}_n) \left[Q'_{n2}(Q_{s1}) + \bar{S}'_n(Q_{s1}) \right] - u''_n(Q_{s1} - \bar{S}_n) \left[1 - \bar{S}'_n(Q_{s1}) \right] \right\} \\ &\quad - \psi u''_n(\bar{S}_n) \left[Q'_{n2}(Q_{s1}) + \bar{S}'_n(Q_{s1}) \right]. \end{aligned}$$

Given that $Q'_{n2}(Q_{s1}) = -1$ and $\bar{S}'_n(Q_{s1}) = 1$ (as implied by (S22) and (S24)), it follows from (S22) that $\partial \Pi / \partial Q_{s1} |_{Q_{n2}=0} \geq 0$, which means that the firm does not have any incentives to

decrease Q_{s1} in order to get $Q_{n2} > 0$. Thus, it holds $Q_{s2}^{*u} = Q_{n2}^{*u} = 0$. We obtain from consumers' storage decisions (see (S23) and (S24)) that $S_s^{*u} = \bar{S}_n^{*u} = Q_{s1}^{*u}/2$, which yields $\Psi_n^{*u} = 2[u_n(Q_{s1}^{*u}/2) - u_s(Q_{s1}^{*u}/2)]$. As $u'_s(Q_{s1}^m) - \gamma = 0$ and $u'_n(Q_{s1}^{*u}/2) - \gamma \leq 0$ (see (S22)), we find from $u'_n(\cdot) > u'_s(\cdot)$ that $Q_{s1}^{*u}/2 > Q_{s1}^m$, which implies $u_n(Q_{s1}^{*u}/2) - u_s(Q_{s1}^{*u}/2) > u_n(Q_{s1}^m) - u_s(Q_{s1}^m)$. Furthermore, it follows from $Q_{s2}^m < Q_{s1}^m < Q_{s1}^{*u}/2$ that $u_n(Q_{s1}^{*u}/2) - u_s(Q_{s1}^{*u}/2) > u_n(Q_{s2}^m) - u_s(Q_{s2}^m)$ (recall $u'_n(\cdot) > u'_s(\cdot)$). Thus, it holds $\Psi_n^{*u} > \Psi_n^m$.

Now, we turn to the case of naïve overestimating consumers, where $u'_s(\cdot) > u'_n(\cdot)$. As all consumers believe that they have the same preferences at first period purchase stage and naïfs discover lower valuation for the good, sophisticates consume (weakly) more than naïfs in the first period and thus their ex post storage is (weakly) lower, i.e., $\bar{S}_s \leq \bar{S}_n$. As with underestimating consumers, it can be shown that the participation constraint (S18) for naïfs and the incentive constraint (S19) for sophisticates are binding in equilibrium, whereas the participation constraint (S17) for sophisticates and the incentive constraint (S18) for naïfs are slack. Substituting the binding constraints (S18) and (S19) into the firm's second period maximization problem in (S16) and taking the first-order conditions for $Q_{s2} \geq 0$ and $Q_{n2} \geq 0$ we obtain

$$u'_s(Q_{s2} + \bar{S}_s) - \gamma \leq 0 \quad (\text{S29})$$

and

$$u'_n(Q_{n2} + \bar{S}_n) - \lambda u'_s(Q_{n2} + \bar{S}_s) - (1 - \lambda)\gamma \leq 0, \quad (\text{S30})$$

where the equalities hold for $Q_{s2} > 0$ and $Q_{n2} > 0$, respectively. In equilibrium, it holds $Q_{s2} \geq Q_{n2}$. Hence, given the first period pricing policy and consumers' storage decisions, the second period pricing policy is $\{(Q_{s2}, P_{s2}), (Q_{n2}, P_{n2})\}$, where Q_{s2} and Q_{n2} respectively satisfy (S29) and (S30), whereas P_{s2} and P_{n2} follow from the binding constraints (S18) and (S19). It holds $Q_{n2} = P_{n2} = 0$ for $\lambda \geq [u'_n(\bar{S}_n) - \gamma] / [u'_s(\bar{S}_s) - \gamma]$.

In anticipation of the second period pricing policy $\{(Q_{s2}, P_{s2}), (Q_{n2}, P_{n2})\}$ (as implied by (S16) with $u'_s(\cdot) > u'_n(\cdot)$) and consumers' storage decisions (as implied by (S23) and (S24)), the firm's first period maximization problem is given by (S25) subject to the participation constraint (S26) and the non-skipping constraint (S27), which is binding in equilibrium (as before). Taking the first-order condition for Q_{s1} yields after some manipulation

$$\begin{aligned} & u'_s(Q_{s1} - S_s) + Q'_{s2}(Q_{s1}) [u'_s(Q_{s2} + S_s) - u'_s(Q_{s2})] + S'_s(Q_{s1}) [u'_s(Q_{s2} + S_s) - u'_s(Q_{s1} - S_s) \\ & - c] + \lambda \{u'_s(Q_{s2} + S_s) [Q'_{s2}(Q_{s1}) + S'_s(Q_{s1})] - u'_s(Q_{n2} + S_s) [Q'_{n2}(Q_{s1}) + S'_s(Q_{s1})] \\ & + u'_n(Q_{n2} + \bar{S}_n) [Q'_{n2}(Q_{s1}) + \bar{S}'_n(Q_{s1})] - u'_n(\bar{S}_n) \bar{S}'_n(Q_{s1}) - \gamma Q'_{s2}(Q_{s1})\} \\ & + (1 - \lambda) \{u'_n(Q_{n2} + \bar{S}_n) [Q'_{n2}(Q_{s1}) + \bar{S}'_n(Q_{s1})] - u'_n(\bar{S}_n) \bar{S}'_n(Q_{s1}) - \gamma Q'_{n2}(Q_{s1})\} \\ & - \mu \{u''_s(Q_{s2} + S_s) [Q'_{s2}(Q_{s1}) + S'_s(Q_{s1})] - u''_s(Q_{s1} - S_s) [1 - S'_s(Q_{s1})]\} \\ & - \nu \{u''_n(Q_{n2} + \bar{S}_n) [Q'_{n2}(Q_{s1}) + \bar{S}'_n(Q_{s1})] - u''_n(Q_{s1} - \bar{S}_n) [1 - \bar{S}'_n(Q_{s1})]\} \\ & - \varphi u''_s(Q_{s2} + S_s) [Q'_{s2}(Q_{s1}) + S'_s(Q_{s1})] - \psi \{u''_n(Q_{n2} + \bar{S}_n) [Q'_{n2}(Q_{s1}) + \bar{S}'_n(Q_{s1})] \\ & - \lambda u''_s(Q_{n2} + S_s) [Q'_{n2}(Q_{s1}) + S'_s(Q_{s1})]\} - \gamma = 0, \quad (\text{S31}) \end{aligned}$$

where $\mu \geq 0$ and $\nu \geq 0$ represent the Kuhn-Tucker multipliers associated with consumers' storage decisions (as implied by (S23) and (S24)), and $\varphi \geq 0$ and $\psi \geq 0$ denote the Kuhn-Tucker multipliers associated with the first-order conditions (S29) and (S30). Consequently, in the presence of naïve overestimating consumers, the pricing policy in the dynamic storage solution is given by (i) the first period contract $(Q_{s1}^{*o}, P_{s1}^{*o})$, where Q_{s1}^{*o} and P_{s1}^{*o} respectively satisfy the first-order condition (S31) and the binding non-skipping constraint (S27), and (ii) the second period contract menu $\{(Q_{s2}^{*o}, P_{s2}^{*o}), (Q_{n2}^{*o}, P_{n2}^{*o})\}$, which stems from the firm's maximization problem in (S16) (where $u'_s(\cdot) > u'_n(\cdot)$), with the associated first-order conditions (S29) and (S30). The equilibrium levels of storage are given by S_s^{*o} for sophisticated consumers and \bar{S}_n^{*o} for naïve consumers (as implied by (S23) and (S24)). The dynamic storage solution with naïve overestimating consumers requires that (at least) naïfs engage in storage activities, i.e., $\bar{S}_n^{*o} > 0$, which emerges for sufficiently small values of the storage cost, i.e., $c < u'_n(Q_{n2}|\bar{S}_n=0) - u'_n(Q_{s1}|\bar{S}_n=0)$ (see (S24)).

Using the binding constraint (S27), we find that sophisticated consumers obtain the following surplus

$$\begin{aligned}\Psi_s^{*o} &= u_s(Q_{s2}^{*o}) - P_{s2}^{*o} \\ &= u_s(Q_{s2}^{*o}) + u_s(Q_{n2}^{*o} + S_s^{*o}) + u_n(\bar{S}_n^{*o}) \\ &\quad - u_s(Q_{s2}^{*o} + S_s^{*o}) - u_n(Q_{n2}^{*o} + \bar{S}_n^{*o}) \geq 0,\end{aligned}$$

where the equality holds if and only if $Q_{s2}^{*o} = P_{s2}^{*o} = 0$. Naïve consumers obtain

$$\begin{aligned}\Psi_n^{*o} &= u_n(Q_{s1}^{*o} - \bar{S}_n^{*o}) - c\bar{S}_n^{*o} - P_{s1}^{*o} + u_n(Q_{n2}^{*o} + \bar{S}_n^{*o}) - P_{n2}^{*o} \\ &= -u_s(Q_{s1}^{*o} - S_s^{*o}) - u_s(Q_{s2}^{*o} + S_s^{*o}) + u_s(Q_{s1}^{*o} - \bar{S}_n^{*o}) \\ &\quad + u_s(Q_{s2}^{*o}) + u_n(\bar{S}_n^{*o}) - c(\bar{S}_n^{*o} - S_s^{*o}) \leq 0.\end{aligned}$$

In the static solution, the pricing policy is given by (i) the first period contract (Q_{s1}^m, P_{s1}^m) , where Q_{s1}^m is such that $u'_s(Q_{s1}^m) - \gamma = 0$ and $P_{s1}^m = u_s(Q_{s1}^m)$, and (ii) the second period contract menu $\{(Q_{s2}^m, P_{s2}^m), (Q_{n2}^m, P_{n2}^m)\}$, which stems from the firm's maximization problem in (S16) (where $u'_s(\cdot) > u'_n(\cdot)$), with the associated first-order conditions (S29) and (S30) for $\bar{S}_s = \bar{S}_n = 0$. The static solution can only emerge in the absence of storage, which occurs for sufficiently large values of the storage cost, i.e., $c \geq u'_n(Q_{n2}|\bar{S}_n=0) - u'_n(Q_{s1}|\bar{S}_n=0)$ (see above). We find that sophisticated consumers obtain $\Psi_s^m = u_s(Q_{n2}^m) - u_n(Q_{n2}^m) \geq 0$ and naïve consumers obtain $\Psi_n^m = -u_s(Q_{s1}^m) + u_n(Q_{s1}^m) < 0$.

We now compare the dynamic storage solution and the static solution in terms of consumer surplus. For the sake of convenience, we confine our attention to the case where the share of sophisticated consumers is large enough, i.e., $\lambda \geq \tilde{\lambda}_{nl}^o \triangleq [u'_n(\bar{S}_n^{*o}) - \gamma] / [u'_s(\bar{S}_s^{*o}) - \gamma]$ (see the discussion after (S30)). This implies that $Q_{n2}^{*o} = P_{n2}^{*o} = 0$. We show that $\Psi_s^{*o} \leq \Psi_s^m$ and $\Psi_n^{*o} < \Psi_n^m$, provided that the storage cost c is low enough. To proceed in a insightful manner, we fix $c = 0$. Our results carry over to the case where c is low enough. First, we demonstrate that the firm cannot benefit from $Q_{s2} > 0$. It follows from the first-order condition for Q_{s1} in (S31) that the impact of Q_{s1} on the firm's profit Π through Q_{s2} evaluated at $Q_{s2} = 0$ can be

written as

$$\begin{aligned} \frac{\partial \Pi}{\partial Q_{s1}} \Big|_{Q_{s2}=0} &= u'_s(Q_{s1} - S_s) + Q'_{s2}(Q_{s1}) [u'_s(S_s) - u'_s(0)] + S'_s(Q_{s1}) [u'_s(S_s) - u'_s(Q_{s1} - S_s)] \\ &+ \lambda \{ u'_s(S_s) [Q'_{s2}(Q_{s1}) + S'_s(Q_{s1})] - u'_s(S_s) S'_s(Q_{s1}) - \gamma Q'_{s2}(Q_{s1}) \} \\ &- \mu \{ u''_s(Q_{s2} + S_s) [Q'_{s2}(Q_{s1}) + S'_s(Q_{s1})] - u''_s(Q_{s1} - S_s) [1 - S'_s(Q_{s1})] \} \\ &- \varphi u''_s(Q_{s2} + S_s) [Q'_{s2}(Q_{s1}) + S'_s(Q_{s1})]. \end{aligned}$$

We find from consumers' storage decisions (see (S23) and (S24)) that S_s is such that the expression in the last square brackets in the first line vanishes, which implies $S_s = Q_{s1}/2$. As $Q'_{s2}(Q_{s1}) = -1$ and $S'_s(Q_{s1}) = 1$ (as implied by (S23) and (S29)), it follows from (S29) that $\partial \Pi / \partial Q_{s1} |_{Q_{s2}=0} > 0$, which means that the firm does not have any incentives to decrease Q_{s1} in order to get $Q_{s2} > 0$. Thus, it holds $Q_{s2}^{*o} = Q_{n2}^{*o} = 0$. As $\Psi_s^{*o} = 0$, we have $\Psi_s^{*o} \leq \Psi_s^m$. Furthermore, we obtain from consumers' storage decisions (see (S23) and (S24)) that $S_s^{*o} = \bar{S}_n^{*o} = Q_{s1}^{*o}/2$, which yields $\Psi_n^{*o} = -2[u_s(Q_{s1}^{*o}/2) - u_n(Q_{s1}^{*o}/2)] < 0$. As $u'_s(Q_{s1}^m) - \gamma = 0$ and $u'_s(Q_{s1}^{*o}/2) - \gamma \leq 0$ (see (S29)), we find that $Q_{s1}^{*o}/2 \geq Q_{s1}^m$, which implies that $u_s(Q_{s1}^{*o}/2) - u_n(Q_{s1}^{*o}/2) \geq u_s(Q_{s1}^m) - u_n(Q_{s1}^m)$ (recall $u'_s(\cdot) > u'_n(\cdot)$). Thus, it holds $\Psi_n^{*o} < \Psi_n^m < 0$. ■

5 Price commitment

To investigate the role of firm's price commitment, we consider the situation where the firm is able to commit to future prices. In the following remark, we establish the main features of the dynamic storage solution under full commitment, according to whether naïve consumers underestimate or overestimate their preferences.

Remark 5 A. *With naïve underestimating consumers, under full commitment, the dynamic storage solution arises in equilibrium for $c < \hat{c}^{cu}$. This yields prices p_1^{cu} and $p_2^{cu} = p_1^{cu} + c$ as well as ex ante storage $S^{cu} > 0$ and ex post storage $\bar{S}^{cu} = \lambda S^{cu} > 0$.*

B. With naïve overestimating consumers, under full commitment, the dynamic storage solution arises in equilibrium for $c < \bar{c}^{co}$. This yields prices p_1^{co} and p_2^{co} , with $p_1^{co} > p_2^{co}$, as well as ex post storage $\bar{S}^{co} > 0$.

Proof of Remark 5. First, we consider the case of naïve underestimating consumers. Following the same rationale as in the proof of Lemma 1 of the paper, the firm does not have any incentives to promote ex post storage by naïve consumers. As storage can emerge only if the storability constraint is binding, i.e., $p_1 + c = p_2$, the firm's profit maximization problem writes as

$$\max_{p_1} p_1 [D_s(p_1) + S(p_1)] + (p_1 + c) [D_2(p_1 + c) - \lambda S(p_1)],$$

where $D_2(p_2) \triangleq \lambda D_s(p_2) + (1 - \lambda) D_n(p_2)$. The firm's profit increases with $S(\cdot)$ if and only if $p_1(1 - \lambda) - c\lambda > 0$. In this case, the firm prefers to promote the level of storage that covers the second period demand, i.e., $S(p_1) = D_2(p_1 + c)$. Otherwise, no storage emerges in

equilibrium. For $p_1(1 - \lambda) - c\lambda > 0$, the firm's maximization problem reduces to

$$\max_{p_1} p_1 D_s(p_1) + [(2 - \lambda)p_1 + (1 - \lambda)c] D_2(p_1 + c).$$

The first-order condition for p_1 is

$$D_s(p_1) + p_1 D'_s(p_1) + (2 - \lambda) D_2(p_1 + c) + [(2 - \lambda)p_1 + (1 - \lambda)c] D'_2(p_1 + c) = 0. \quad (\text{S32})$$

The equilibrium full commitment prices with storage are given by

$$p_1^{cu} = -\frac{D_s(p_1^{cu}) + (2 - \lambda) D_2(p_1^{cu} + c) + (1 - \lambda) D'_2(p_1^{cu} + c) c}{D'_s(p_1^{cu}) + (2 - \lambda) D'_2(p_1^{cu} + c)} \quad \text{and} \quad p_2^{cu} = p_1^{cu} + c. \quad (\text{S33})$$

The equilibrium full commitment levels of ex ante and ex post storage are $S^{cu} \triangleq S(p_1^{cu}) = D_2(p_1^{cu} + c)$ and $\bar{S}^{cu} = \lambda S^{cu} = \lambda D_2(p_1^{cu} + c)$, respectively. As $p_1^{cu}(1 - \lambda) - c\lambda > 0$ for c low enough, there exists a threshold \hat{c}^{cu} such that for $c < \hat{c}^{cu}$ the dynamic storage solution arises in equilibrium. In line with the proof of Lemma 1 of the paper, we find that, if at the prices in (S33) naïfs store ex post, the firm prefers to set higher prices that mitigate the distortion from the benchmark case of perfectly aware consumers. The equilibrium full commitment prices become \tilde{p}_1^{cu} and $\tilde{p}_2^{cu} = \tilde{p}_1^{cu} + c$, where $\tilde{p}_\tau^{cu} > p_\tau^{cu}$, $\tau \in \{1, 2\}$. Specifically, \tilde{p}_1^{cu} is such that only sophisticates store ex post, i.e., $\bar{S}(\tilde{p}_1^{cu}) = \lambda S(\tilde{p}_1^{cu}) = \lambda D_2(\tilde{p}_1^{cu} + c)$, and naïfs' marginal utilities are equalized over time, i.e., $u'_n(D_s(\tilde{p}_1^{cu}) + s(\tilde{p}_1^{cu})) = \tilde{p}_1^{cu}$.

Now, we turn to the case of naïve overestimating consumers. As $p_2 = p_2^e$, the firm's profit maximization problem writes as

$$\max_{\{p_1, p_2\}} p_1 D_s(p_1) + p_2 [D_2(p_2) - \bar{S}(p_1, p_2)],$$

where $\bar{S}(p_1, p_2) = (1 - \lambda) \bar{s}_n(p_1, p_2)$ and $\bar{s}_n(p_1, p_2) = D_s(p_1) - D_n(p_2 - c)$, as implied by the condition for the equalization of intertemporal marginal utilities in equation (3) of the paper (for $s(\cdot) = 0$ and $k = l$). Taking the first-order conditions for p_1 and p_2 yields

$$D_s(p_1) + p_1 D'_s(p_1) - (1 - \lambda) p_2 D'_s(p_1) = 0 \quad (\text{S34})$$

and

$$D_2(p_2) + p_2 [D'_2(p_2) + (1 - \lambda) D'_n(p_2 - c)] - (1 - \lambda) [D_s(p_1) - D_n(p_2 - c)] = 0. \quad (\text{S35})$$

The equilibrium full commitment prices are p_1^{co} and p_2^{co} that simultaneously satisfy the first-order conditions (S34) and (S35). The equilibrium full commitment ex post storage is $\bar{S}^{co} \triangleq \bar{S}(p_1^{co}, p_2^{co}) = (1 - \lambda) \bar{s}^{co} = (1 - \lambda) [D_s(p_1^{co}) - D_n(p_2^{co} - c)]$. As $\lim_{\lambda \rightarrow 1} p_1^{co} = \lim_{\lambda \rightarrow 1} p_2^{co}$, we have $\bar{S}^{co} > 0$ for λ high enough and c low enough. Thus, there exists a threshold \bar{c}^{co} such that the dynamic storage solution arises in equilibrium for $c < \bar{c}^{co}$. Substituting p_1^{sm} (derived in the proof of Lemma 1 of the paper) into the left-hand side of the first-order condition for p_1^{co} in (S34) yields $-(1 - \lambda) p_2 D'_s(p_1^{sm}) > 0$, which implies $p_1^{co} > p_1^{sm}$. Substituting p_2^{sm} (derived in

the proof of Lemma 1 of the paper) into the left-hand side of the first-order condition for p_2^{co} in (S35) yields $-(1 - \lambda) [D_s(p_1) - D_n(p_2^m - c) - p_2 D_n'(p_2^m - c)] < 0$, which implies $p_2^{co} < p_2^m$. As $p_1^{sm} > p_2^m$, we obtain that $p_1^{co} > p_2^{co}$. ■

In the following remark, we compare the equilibrium prices under full commitment and limited commitment in the most relevant case where the storage cost is sufficiently low that storage emerges in equilibrium irrespective of the commitment regime. To this aim, with naïve underestimating consumers, it is helpful to introduce the difference in the firm's gains from storage between full and limited commitment for a given price p_1 , denoted by $\Gamma(p_1) \triangleq [p_1(1 - \lambda) - c\lambda] [S^{cu}(p_1) - S^{*u}(p_1)]$.⁸

Remark 6 *A. With naïve underestimating consumers, in the presence of storage, the equilibrium prices under full commitment are higher than under limited commitment, $p_\tau^{cu} > p_\tau^{*u}$, $\tau \in \{1, 2\}$, if and only if the difference in the firm's gains from storage between full and limited commitment increases with the price in equilibrium, $\partial\Gamma(p_1^{cu})/\partial p_1 > 0$.*

B. With naïve overestimating consumers, in the presence of storage, under full commitment the firm has incentives to charge a higher first period price and a lower second period price in comparison with limited commitment.

Proof of Remark 6. First, we consider the case of naïve underestimating consumers. Substituting the first-order condition for p_1^{cu} in (S32) into the left-hand side of the first-order condition for p_1^{*u} in (A6) of the paper yields after some manipulation

$$\begin{aligned} & \frac{1 - \lambda}{\lambda} [(1 - \lambda) D_2(p_1^{cu} + c) + (p_1^{cu} + c) D_2'(p_1^{cu} + c)] \\ & + \frac{p_1^{cu}(1 - \lambda) - c\lambda}{\lambda} [(2 - \lambda) D_2'(p_1^{cu} + c) + (p_1^{cu} + c) D_2''(p_1^{cu} + c)] \\ & = -\frac{\partial\Gamma(p_1^{cu})}{\partial p_1}. \end{aligned}$$

Thus, it holds $p_\tau^{cu} > p_\tau^{*u}$, $\tau \in \{1, 2\}$, if and only if $\partial\Gamma(p_1^{cu})/\partial p_1 > 0$. Note that $S^{*u}(p_1) = [D_2(p_1 + c) + (p_1 + c) D_2'(p_1 + c)]/\lambda < S^{cu}(p_1) = D_2(p_1 + c)$ (as storage cannot exceed the second period demand), where $S^{*u}(p_1)$ and $S^{cu}(p_1)$ are respectively derived in the proofs of Lemma 1 of the paper and of Remark 5. This implies that the expression in the first square brackets is negative. Hence, as $p_1^{cu}(1 - \lambda) - c\lambda > 0$, a sufficient (albeit not necessary) condition for $\partial\Gamma(p_1^{cu})/\partial p_1 > 0$ is that the expression in the second square brackets is non-positive, which occurs if and only if $\partial S^{*u}(p_1^{cu})/\partial p_1 \leq \partial S^{cu}(p_1^{cu})/\partial p_1$. This is satisfied when $D_2(\cdot)$ is not too convex. Consequently, we have $p_\tau^{cu} > p_\tau^{*u}$, $\tau \in \{1, 2\}$, as long as $D_2(\cdot)$ is not too convex.

Now, we move to the case of naïve overestimating consumers. Substituting $p_1^{cu}(p_2)$ in the first-order condition (S34) into the left-hand side of the first-order condition for p_1^{*o} in (A11) of the paper for a given p_2 yields $p_2 [(1 - \lambda) D_s'(p_1^{cu}(p_2)) - d\bar{S}^{*o}(p_1^{cu}(p_2), p_2)/dp_1] < 0$, where the inequality follows from the expression (A12) of the paper.⁹ Furthermore, substituting

⁸For the sake of clarity, with a small abuse of notation we identify by $S^{cu}(p_1)$ and $S^{*u}(p_1)$ the levels of storage as a function of p_1 under full commitment and limited commitment, respectively.

⁹In line with the proof of Remark 5, for the sake of clarity, with a small abuse of notation we define by $p_1^{cu}(p_2)$ the price under full commitment as a function of p_2 (and $p_2^{cu}(p_1)$ mutatis mutandis). We also denote by $\bar{S}^{*o}(p_1, p_2)$ the level of ex post storage under limited commitment as a function of p_1 and p_2 .

$p_2^{cu}(p_1)$ in the first-order condition (S35) into the left-hand side of the first-order condition for p_2^{*o} in (A9) of the paper for a given p_1 , we find that $-(1-\lambda)p_2^{cu}(p_1)D'_n(p_2^{cu}(p_1)-c) > 0$. Consequently, for any given second (first) period price, under full commitment the firm has incentives to charge a higher (lower) price in the first (second) period in comparison with limited commitment. Using linear demand functions (with parallel shift), we obtain that $p_1^{co} > p_1^{*o}$ and $p_2^{co} < p_2^{*o}$. ■

6 Data collection and further estimations

In the empirical application of our model (see Section 7 of the paper), we collect Amazon and third party prices available in graphical form in Camelcamelcamel, for different storable product categories, namely, vitamin B, vitamin C, multivitamin, soup, tuna, and toilet paper in the United States from January 2010 to June 2024. We refer to the paper (Section 7) for the investigation of vitamins. Soup and tuna have been traditionally found to exhibit seasonal fluctuations in demand (with peaks during the winter and Lent period, respectively) and countercyclical pricing (e.g., Butters et al. 2025; Chevalier 2003; Haviv 2022; Kwon et al. 2023; Nevo and Hatzitaskos 2006). We also include toilet paper because, despite some degree of demand seasonality (e.g., Butters et al. 2025), its consumption is likely to be relatively stable. We hand-match the prices for these product categories and transform them into actual prices through a computer vision assisted software.¹⁰ The collected prices are converted into daily and monthly prices as follows. As for daily prices, whenever there are missing values across days, we consider the last available one. When there are multiple prices within the day, we compute the minimum value when the price drops and the maximum value when the price increases as these are the actual values. Monthly prices are simply the average prices of the month. Moreover, we collect data on searches through Google Trends. These are yearly data so that weekly searches can be downloaded. Data are comparable over the years as they are scaled up to a maximum value of 100. This allows us to control for additional incoming Internet users over the years that are possibly not correlated with increasing demand for the product. Table S1 (see below) provides summary statistics.

As shown in Table S2 (see below), we find seasonality in demand for all products, using searches through Google Trends (daily data). We also present the results of a t-test for difference in means (Welch's test for unequal variances) in terms of prices between high and low demand season. In the aggregate market for vitamins, the existence of price differences between high and low demand season is statistically significant at the 1% level. The aggregate mean price appears to be about 1.5% higher during low demand season compared to high demand season (\$18.67 vs. \$18.41), which indicates the presence of countercyclical pricing. We find that prices for each vitamin product are countercyclical, with the price difference being statistically significant at the 1% level for vitamin C and multivitamin but not significant for vitamin B. Prices for soup are also countercyclical but their difference is neither statistically nor economically significant. Tuna follows a procyclical price behavior (at the 5% significance level). Prices for toilet paper are also procyclical but the results are not statistically significant.

¹⁰The software is available at <https://automeris.io/posts/> (last retrieved in June 2025).

We consider the following specification of the OLS regression for soup, tuna, and toilet paper

$$P_{ibt} = \alpha_0 + \alpha_1 Covid + \alpha_2 High + \alpha_3 Direct + \alpha_4 Covid \times High + \alpha_5 Direct \times High + v_{ibt}. \quad (S36)$$

The regression in (S36) is similar to the one for the vitamin market (see Section 7 of the paper). Again, the variable *Direct* is dropped due to multicollinearity. The results are presented in Table S3 (see below). Prices for soup are countercyclical but, once we account for Covid-19 that has inflated demand in winter (as attested by searches through Google Trends), price procyclicality emerges, which is consistent with our theoretical findings. Moving to tuna, we find that, when high demand season is restricted to Lent (see columns (3) and (4)), prices are procyclical, possibly due to a significant demand increase (as implied by searches through Google Trends). Such positive effect weakens when we extend high demand season (see columns (5) and (6)). Consistently with our model, demand volatility (as measured by standard deviation in searches in Table S1) is higher when high demand season only covers Lent. Lastly, we consider toilet paper as its consumption is arguably constant over time, which implies that prices are unlikely to be manipulated in order to stimulate consumption through storage. In line with our presumption, we do not find significant differences in prices (either economically or statistically). The only statistically significant effect is the one stemming from Covid-19, perhaps because of panic buying. In this case, price increases can be explained by the traditional theory that predicts higher prices associated with higher demand.

Table S1: Summary statistics.

Variable	Obs	Mean	Std. Dev.	Min	Max
Vitamins					
Year	15			2010	2024
Price	51,038	18.546	12.132	0.006	169.32
Log_price	51,038	2.735	0.626	-5.155	5.132
Vit.C	13,999				
Direct	20,270				
Highdemand	26,994				
brand	481	0	481		
Soup					
Year	15			2010	2024
Price	7,295	21.763	13.781	1.448	84.644
Log_price	7,295	2.850	0.729	0.370	4.438
Direct	6,678				
Highdemand	4,914				
brand	98	0	97		
Tuna					
Year	15			2010	2024
Price	9,848	18.546	12.132	0.006	169.32
Log_price	9,848	2.668	0.964	-1.197	4.861
Direct	8,550				
Highdemand	4,224				
brand	96	1	96		
Toilet Paper					
Year	15			2010	2024
Price	9,451	40.757	20.280	0.458	144.455
Log_price	9,451				
Direct	6,498				
Highdemand	8,811				
brand	89	1	89		

Table 2: Table S2: Seasonality in Google searches, prices, difference in means, t-tests.

	Mean Google Search	t-test for Searches (p-value)	Standard Deviation for Searches	Mean Price (\$)	t-test for Prices (p-value)	Standard Deviation Prices
Vitamin C						
Low Demand	69.95	0.000	12.43	14.78	0.000	8.29
High Demand	81.61		14.89	14.67		8.24
Multivitamin						
Low Demand	80.30	0.000	5.82	22.60	0.000	13.38
High Demand	83.32		9.13	22.45		13.49
Vitamin B						
Low Demand	76.64	0.000	8.83	17.09	0.184	12.20
High Demand	84.27		7.63	17.04		12.28
Vitamins						
Low Demand	Same as Vitamin C and Multivitamin			18.67	0.000	12.23
High Demand				18.41		12.29
Soups						
Low Demand	41.59	0.000	9.10	22.64	0.892	14.25
High Demand	71.01		15.68	22.63		14.42
Tuna						
Low Demand	66.86	0.000	13.41	21.58	0.022	21.48
High Demand	76.14		11.71	21.79		21.70
Toilet Paper						
Low Demand	67.19	0.000	21.91	39.77	0.148	39.67
High Demand	72.02		20.88	39.89		39.75

Table 3: Table S3: OLS regressions for soup, tuna, and toilet paper.

The table reports coefficient estimates and t-statistics (in parentheses) from estimations for soup, tuna, and toilet paper. The sample period is January 2010-June 2024. The dependent variable is the natural logarithm of price. Estimation method is OLS with the fixed effects reported in the lower part of the table and robust standard errors clustered by brand. Columns (1) and (2) refer to soup, columns (3) and (4) to tuna where High is the Lent period, columns (5) and (6) to tuna where High is the extended period (according to the definition in Section 7 of the paper), and columns (7) and (8) to toilet paper. The lower part of the table also reports the number of observations, the number of brands, and the adjusted R-squared. The ***, **, and * marks denote statistical significance at the 1%, 5%, and 10% level, respectively.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Covid	0.033 (1.44)	0.035 (1.56)	0.080*** (5.02)	0.082*** (5.32)	0.077*** (5.38)	0.077*** (5.39)	0.149*** (4.10)	0.150*** (4.10)
High	-0.046*** (-3.08)	-0.039*** (-2.90)	0.026** (2.30)	0.017** (2.42)	0.013 (1.60)	0.016** (2.17)	0.002 (0.16)	0.001 (0.11)
Covid×High	0.046*** (4.21)	0.039*** (4.27)	-0.016 (-1.37)	-0.020** (-2.02)	-0.013 (-1.33)	-0.015 (-1.53)	-0.005 (-0.58)	-0.006 (-0.72)
Direct×High	0.032** (2.22)	0.024* (1.84)	-0.019* (-1.96)	-0.002 (-1.89)	-0.015 (-1.41)	-0.017* (-1.84)	-0.008 (-0.96)	-0.004 (-0.54)
Constant	2.846*** (77.18)	2.831*** (208.86)	2.556*** (119.45)	2.626*** (336.26)	2.556*** (119.38)	2.631*** (367.22)	3.497*** (54.90)	3.044*** (205.48)
Observations	7,295	7,292	9,848	9,841	9,848	9,841	9,451	9,450
No of Brands	98	98	96	96	96	96	89	89
Brand Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Brand × Year FE	No	Yes	No	Yes	No	Yes	No	Yes
Clustering	Brand	Brand	Brand	Brand	Brand	Brand	Brand	Brand
Adj-R2	0.94	0.97	0.88	0.96	0.88	0.96	0.84	0.93